Micro Hydro Gearbox Design

Final Design Report

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Executive Summary

We were tasked with designing a micro-hydro installation with the goal of creating electricity. The location we selected is in the Nile Creek, near Qualicum Bay, and the expected power output to the generator will be 6.45 kW. The land will need to be acquired from the City of Nanaimo, and an Application for a License to Cut will need to be submitted through the Ministry of Forests for any tree-falling required for the construction of the project. The gearbox was designed with a focus on ease of assembly and there are as few unique parts as possible to reduce machining cost. The method of analysis was mostly trial and error with iteration to converge on acceptable final values. The system contains two helical gearsets, which employ identical input and output shafts, a unique intermediate shaft, and an even gear ratio across both gearsets. The lowest safety factor of the system is on the input shaft, at a value of 1.38. At least one more iteration on shaft size is advised to raise this value to be in the acceptable range of 1.5 to 2. After further iteration, it is expected that the bearings will be the part with the shortest time between service visits at 5.27 years, but currently the lowest service interval is 26 hours, for the main gear. Tolerancing of components was outside the scope of this project, so will need to be completed before manufacturing the gear box. Overall, there is a need for further iteration on the shaft size and the gear service interval, and part tolerancing needs to be finished before the project is complete.

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1.0 Location of Micro Hydro Setup

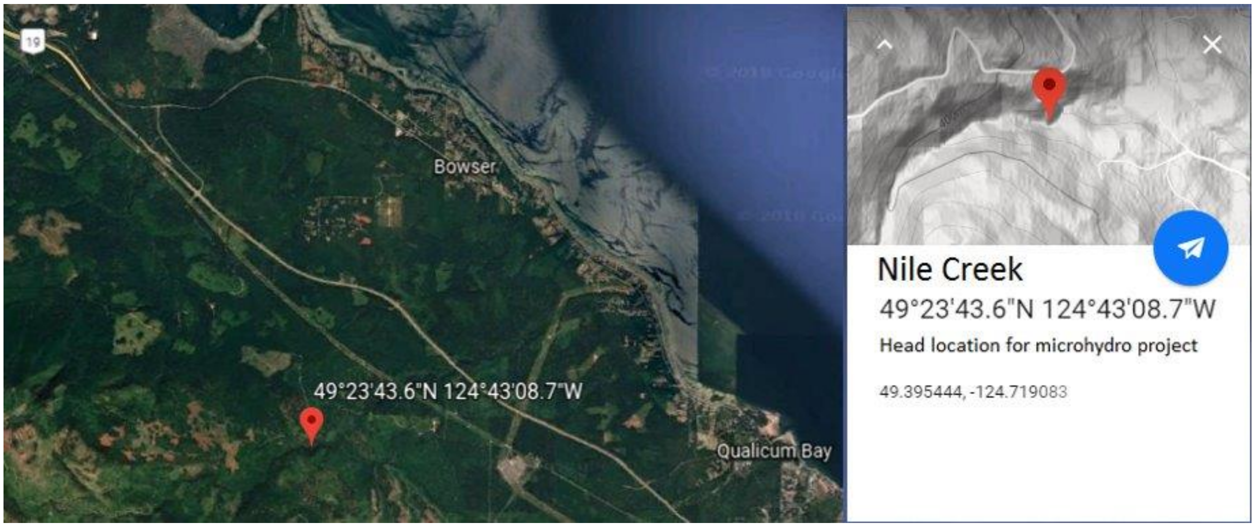


Figure 1: Micro hydro location and relevant information

124º43’08.7”W, as shown in Figure 1. This location is ideal as it has a steep slope, reducing the required length of water diversion to achieve the specified head. Its proximity to two gravel roads and highway 19 will make the site easily accessible and will reduce the maintenance and construction costs significantly, compared to similar projects. Nile Creek also maintains substantial flow rates through the year, ensuring a small to negligible impact of the overall health of the creek. In the low flow months, the water flow is approximately 1.01 m^3/s, which is still far more than the 0.04 m^3/s required to run the turbine. The turbine will generate 6.5kW of electricity with a head of 20 meters.

2.0 Design

The gearbox was designed with one key principle maintained throughout design process: keep it simple. This means minimizing the number of parts required, reduce number of steps to manufacture, and keep the end assembly as easy as possible. The gearbox required a 12.7**:**1 gear ratio with an input torque of 1633Nm, at 37.8 rpm. It was decided to pursue a two-helical gearset gearbox. Each shaft is located using two symmetric tapered roller bearings on each end. This enabled a simple design with one input and output with a single intermediate shaft connecting them, and by evenly distributing the gear ratio between the gear sets, aligned the input and output concentrically. The concentricity of the input and output shafts allows the housing to be machined with a single tool, which decreases machining steps and decreases the chance of the bearing races being misaligned, causing the shaft to bind. To further simplify the assembly and machining process, the input and output shafts were made to be identical, reducing the cost of manufacturing by only requiring a single type of part. Since these two shafts are the same, the secondary shaft was defined to have the same bearing inner race, gear shaft diameter, and shoulder heights. To account for thermal expansion and slight variances in manufacturing we will use shims to preload the compression on the shafts, located between the outer bearing and end cap assemblies. By aligning the gears so that the axial force is directed on to the bearing thrust face, the sustained load on the center section of the shaft was reduced which also minimized likelihood of fatigue failure of the shaft. During the initial design of the gearbox, circlips were used to retain the gears axially on the shafts, but this was altered to be shoulders, when it became apparent that the sustained preload may cause fatigue failure of the circlips. This led to the addition of an endplate to allow for the installation of the smaller gear. It also had the side benefit of decreasing the number of parts, avoided the sharp stress concentration of a circlip groove, and would allow the assembly of the secondary shaft to be inserted as a unit, which simplifies the assembly process. The last feature of note is that the input and output shafts will be connected to the turbine and generator respectively with a jaw coupling. Each shaft will have a key to interface with the jaw, and then the jaws will torsionally connect through a piece of compliant rubber, which will allow for a small amount of shaft misalignment [1] A sample image can be viewed in Figure 2. For example, the CJ series of jaw couplers from the manufacturer Lovejoy would meet our torque and speed requirements. For maintenance, the gear oil can easily be drained by the lower drain valve, and after the gearbox can be refilled by removing the top plug. Overall, this design keeps in line with the key design principle. It requires very few unique parts, has only simple shoulders on the shafts, includes a relatively simple assembly process, and is a relatively modular design that allows for the swapping of gears to change the overall gear ratio if necessary.

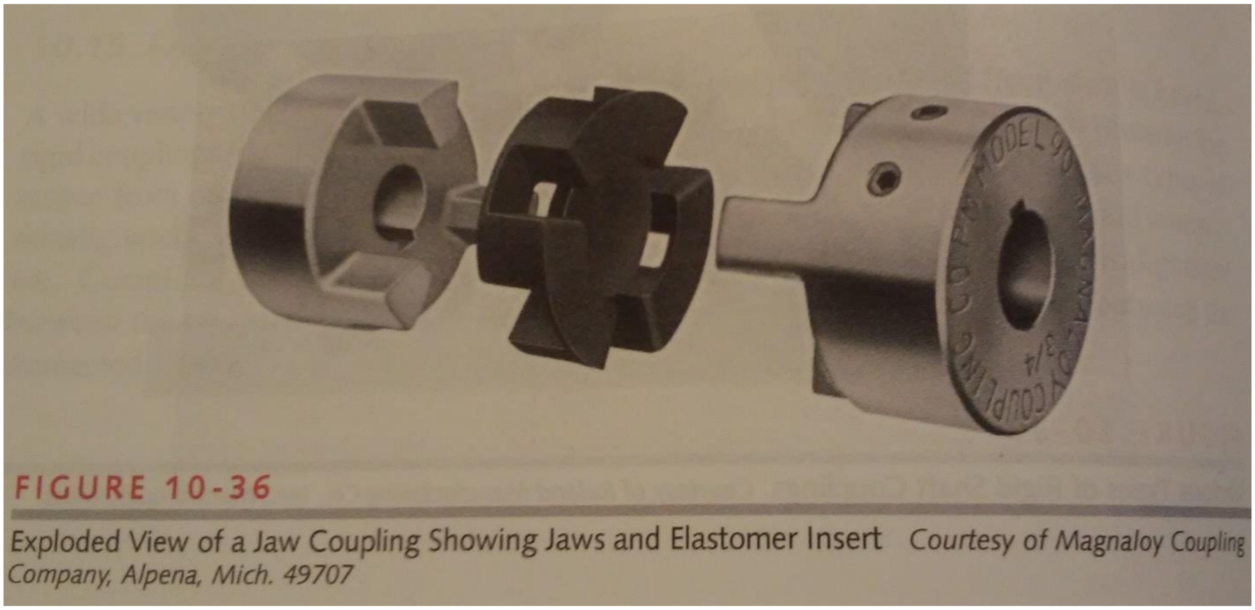


Figure 2. Sample image of a jaw coupling, from [1]

2.1 Assembly

There were several design choices made to ease both maintenance, machinability, and assembly.

- The use of identical gears for both meshes simplify assembly by halving the number of gear types used to prevent confusion, this also reduces cost due to better ability to bulk order and lower inventory costs for spares and replacement parts. This would also simplify maintenance as any technician in the field would have fewer parts to carry with them.
- The use of mirrored input and output shafts have the same effect, despite the output shaft being overbuilt. Most of the cost of the shafts will be from machining, not raw material.

We decided that the cost savings of lowered complexity and higher inter-compatibility outweigh any increase in material cost.

- The use of six identical thrust bearings, keyways, keys, etc. all contribute to the simplicity of the gearbox.
- The output side intermediate shaft endcap is unnecessary for the purposes of assembly. It is there for ease of manufacturing. With a through hole along the whole shaft length, concentricity and alignment of the bearing bores becomes much easier to achieve by using a line hone, then a long rigid tool inserted from the other side of the gearbox. The same logic applies to the bearing housing; it is designed with a through hole make it easier to achieve concentricity and alignment of the bearing bores.
- No welds are used in the production of this gearbox, and all fasteners will be machine screws.

Due to these design choices the cost of this project will be significantly reduced when compared to alternatives, and future costs are minimized.

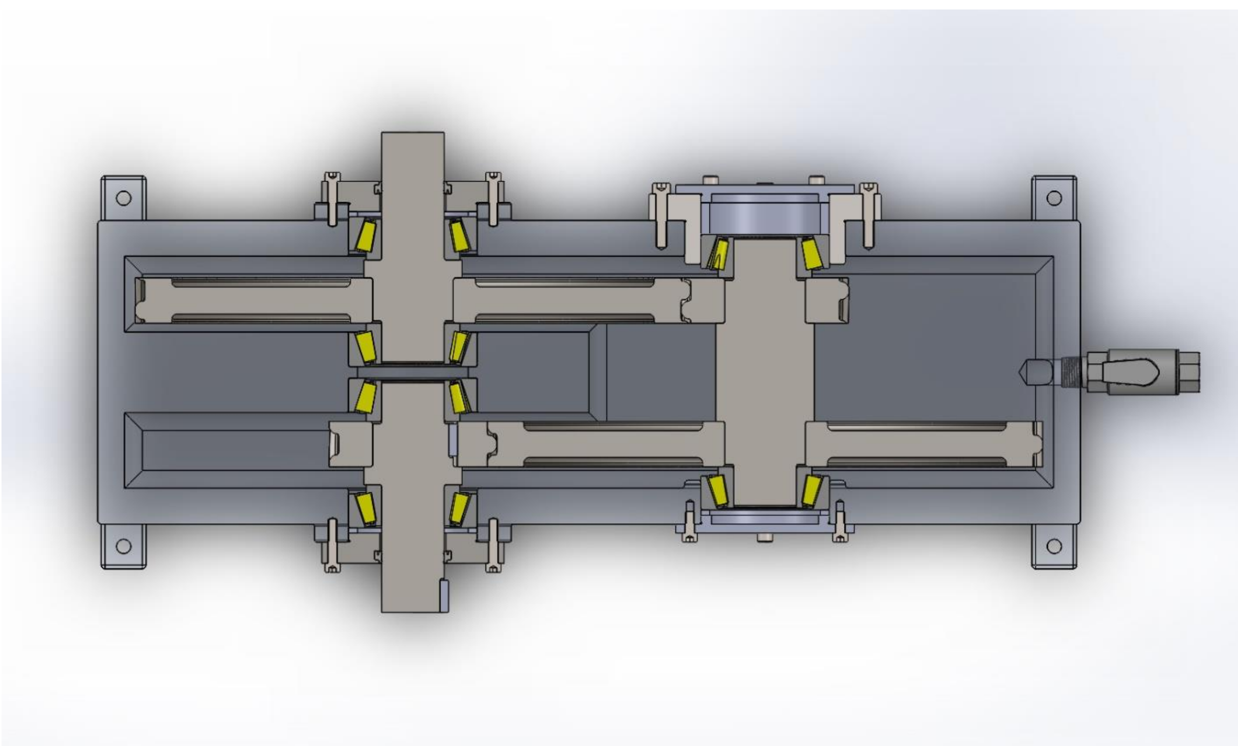


Figure 3: Cross section of assembly

The gearbox is designed to be assembled with the large gears preinserted from the opening in the top of the gearbox, then the various shafts, bearings, and caps are installed via endcaps. A top view of the assembly can be seen in Figure 3. This assembly procedure is as follows:

1. Install two tapered roller bearings in the middle of the gearbox for the input and output shafts, making sure to seat them correctly.
2. Install the wiper seals and O-rings into the endcaps, ensuring proper orientation of the wiper seal.
3. Starting from the input shaft, seat the tapered roller bearing onto the outside of the shaft until it rests against the outer shoulder of the shaft. Then place a key into the gear keyway.
4. Insert the large gear with its teeth sloped away from the intermediate shaft and toward the center of the gearbox. It should be placed into the gearbox from the top, then the input shaft should be slid through its endcap to mate with the gear, then through the preinstalled bearing. Give the end of the shaft a few taps with a soft mallet to ensure proper seating of the gear and bearing before installing the washer shim then the endcap. The proper preload is controlled by the thickness of this shim. The endcap will be held in by 4 circumferential machine screws.
5. Repeat step 3 and 4 with the output shaft and pinion.
6. Install the O-ring into each of the three intermediate shaft endcaps. On the output side insert one of the tapered roller bearings into its bore, then fasten the output endcap to the gearbox. The fasteners should be torqued to 7 N*m for M5, and 12 N*m for M6. As stated above, this endcap exists solely for ease of manufacturing, and its opening does not need to directly contribute to the assembly process.
7. Place the other large gear in line with the pinion on the output shaft with its keyway near the top for visibility. Place both keys into their keyways on the intermediate shaft, then mate the pinion to the shaft on the side that will mesh with the input shaft gear. Then add a bearing, sliding it until it touches the pinion.
8. Take the intermediate shaft assembly, with the pinion on the input side of the shaft, and insert it through the larger opening on the input side. It should pass through the centre of the large gear and into the output side bearing. Then, install the large intermediate shaft endcap to support the output side bearing.

9. Using a tubular spacer to apply the force directly to the outer race of the bearing, and give the spacer a few taps with a soft mallet to ensure proper seating of the gears and bearing before installing the washer shim and the endcap on the input side. Preload of the intermediate shaft is adjusted by the thickness of this shim.
10. With all gear train components in place fill the gearbox with gear oil until the bottom of both large gears are partially submerged, then install the gasket and top cap on the gearbox. This will provide direct lubrication all four gear faces and splash lubrication of all bearings and seals.

3.0 Calculations

The objective of these various calculations was to find the minimum safety factor of the system, then ensure that it is in a suitable range. In this context, with well-known material strengths, the range of 1.5-2 is suitable. In pursuit of favourable factors of safety, many intermediate values were calculated for each component of the assembly. The tables in the sections below enumerate the various stresses of each component in the gearbox.

3.1 Speed and Torque

Table 1. Input values for torque, speed, and power calculations

$N_s = 60$	H = 20 m	$\rho = 997\ kg/m^3$	$Q = 33 \times 10^{-3}\ m^3/s$	g = 9.81 m/s^2	$q_t = 0.7$	$m_g = 12.7$

Using the initial values located in Table 1, and with the use of the equations given by the project description [2], the results in Table 2 were found.

$$Power = \rho g Q H$$

$$Angular\ velocity = \frac{N_s H^{3/4}}{\sqrt{q_t \rho g Q}}$$

$$Input\ torque = \frac{Power}{Angular\ velocity}$$

$$Output\ torque = \frac{Input\ torque}{Gear\ ratio\ m_g}$$

Table 2. Derived torque, speed, and power values

	Torque	Speed	Power
Input	1632.9 N*m	37.8 rpm	6.46 kW
Output	128.6 N*m	479.4 rpm	6.46 kW

3.2 Shaft Specifications

Due to the symmetric design of the input and output shafts, only the input shaft was analyzed for fatigue strength. Given the same geometry but lower forces, the output shaft will have a longer expected lifetime compared to the input shaft, so is irrelevant in the search for the lowest factor of safety of the system. All shafts are made of 4140 Steel Q&T at 400°F with an ultimate tensile strength of 1.772 GPa, and a yield strength of 1.641 GPa.

Each part in a gear box generally has a small amount of clearance radially and axially, to allow for thermal expansion and manufacturability, but not in this design. The amount of radial movement is negligible for all shafts since the bearings will be press-fitted into the housing and should closely fit on the shafts. For axial movement, all bearings are preloaded by the endcaps, and press against the shaft. This limits the axial movement to thermal expansion, which will only serve to increase the preload.

3.2.1 Input Shaft Fatigue Analysis

The stresses in each of the shafts is calculated using the input torque to find steady state forces in locations of the shaft deemed most likely to fail. The points were chosen as they have a stress concentration or is known to have higher mean and alternating stresses. The lowest factor of safety for the gear box was found to be the input coupling keyway on the input shaft, where the keyway removes enough of the cross section to increase the bending stress. The safety factor on this portion was found to be 1.38, which was lowered from its initial 1.55 after realizing that the keyway was the wrong size for the shaft diameter. This point is used in the following calculation.

The first step to calculating to calculate the safety factor was to examine the material and operating conditions to get the corrected endurance strength of the material (Se) using Norton Equation (6.6) [1].

$$(6.6) \quad S_e = C_{load}C_{size}C_{surf}C_{temp}C_{reliab}S_e'$$

$$Axial: S_e = (0.7)(0.82)(0.62)(1.0)(0.81)(886\ MPa) = \ 258\ MPa$$

$$Bending: S_e = (1.0)(0.82)(0.62)(1.0)(0.81)(886\ MPa) = \ 368\ MPa$$

The coefficients $C_{load}, C_{size}, C_{surf}, C_{temp},\ and\ C_{reliab}$ were calculated as per the outlines in chapter 6 of Norton [1].

Then by examining the forces applied to the shaft and then calculating for all reactions, this process starts with a detailed free body diagram as seen by Figure 4. Then with the following simplifying assumptions the initial force-reaction forces can be calculated, the results for the input shaft are outlined in Table 3.

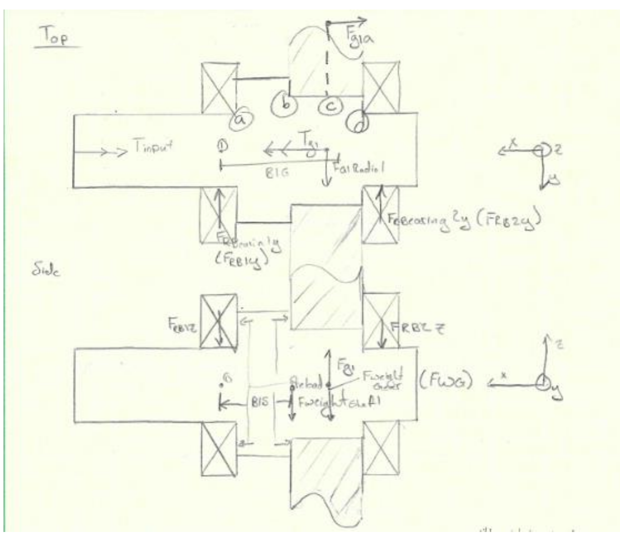


Figure 4: Detailed free body diagram of input shaft

1)The reactions in the bearing happen at the middle point of the bearing.

2) The torque of the gear is transmitted from the center of the gear

3) The axial and radial force from the gear teeth are modeled as directional forces acting from the intersection of the center width of the gear and the pitch circle.

4) The axial force of the gear is transmitted through the bearing directly, but the moment generated by the axial force is still considered

Table 3: Forces from input shaft free body diagram

Force	x-axis	y-axis	z-axis
Fg1 (N)	0.00	0.00	10886.00
Fg1a (N)	2916.89	0.00	0.00

Fg1r (N)	0.00	2916.89	0.00
Fws (N)	0.00	0.00	0.00
Frb2 (N)	0.00	5293.62	7565.41
Frb1 (N)	0.00	-2376.73	3185.44

Following the force-reaction calculations, a solid mechanics analysis can be used to calculate the internal stresses for each of the key locations. This involved sectioning each of the members at the key locations and evaluating the shaft reactions at the cut face, then finding the max value for nominal mean and alternating stresses. The fatigue stress concentration factor (Kf) is then applied to the nominal stresses, with the Kf for alternating stresses defined using Equation (6.11b) and Appendix C of Norton [1]. This process is outlined symbolically for key points a), b), c), and d) in Appendix B-2: Input Shaft Calculations, which determined that the point of failure is at a). Point a) is highlighted with a red line in Figure 5. The numerical results of the stress calculations at this point are presented in Table 4. Note that where the stress on the shaft is constant, such as the axial and torsional loading, $K_f = K_t$, so $q = 1$ in those cases.

$$(6.11b) \quad K_f = 1 + q(K_t - 1)$$

Table 4: Input shaft fatigue data at input keyway

	Nominal Stress	K_t	q	K_f	Cor. Stress
Bending	6.21E+03	1.62	0.92	1.87	1.16E+04
Axial	0.00E+00	1.00	1	1.00	0.00E+00
Torsion	2.19E+08	2.80	1	2.80	6.14E+08

Using the Modified Goodman theory for fatigue failure, we calculated the mean and alternating Von Mises stresses (6.22b), then using equations (6.18 f and g) of Norton [1], we calculated the fatigue safety factor. The final values are given in Table 5 and a full numerical sample can be found in Appendix B-1: Sample Shaft Calculation.

$$(6.22b) \quad \sigma'_a = \sqrt{\sigma_{xa}^2 + \sigma_{ya}^2 - \sigma_{xa}\sigma_{ya} + 3\tau_{xya}^2}$$

$$(6.18f) \quad ZS = \sqrt{(\sigma'_m - \sigma'_{m@S})^2 + (\sigma'_a - \sigma'_{a@S})^2}$$

$$(6.18g) \quad OZ = \sqrt{(\sigma'_m)^2 + (\sigma'_a)^2}$$

$$N_f = \frac{OZ + ZS}{OZ}$$

Table 5. Input shaft Von Mises stress and safety factor

Mean Von Mises stress	6.14E+08
Alternating Von Mises stress	1.16E+04
Line OZ length	6.14E+08
Line ZS length	2.36E+08
Safety factor	**1.38**

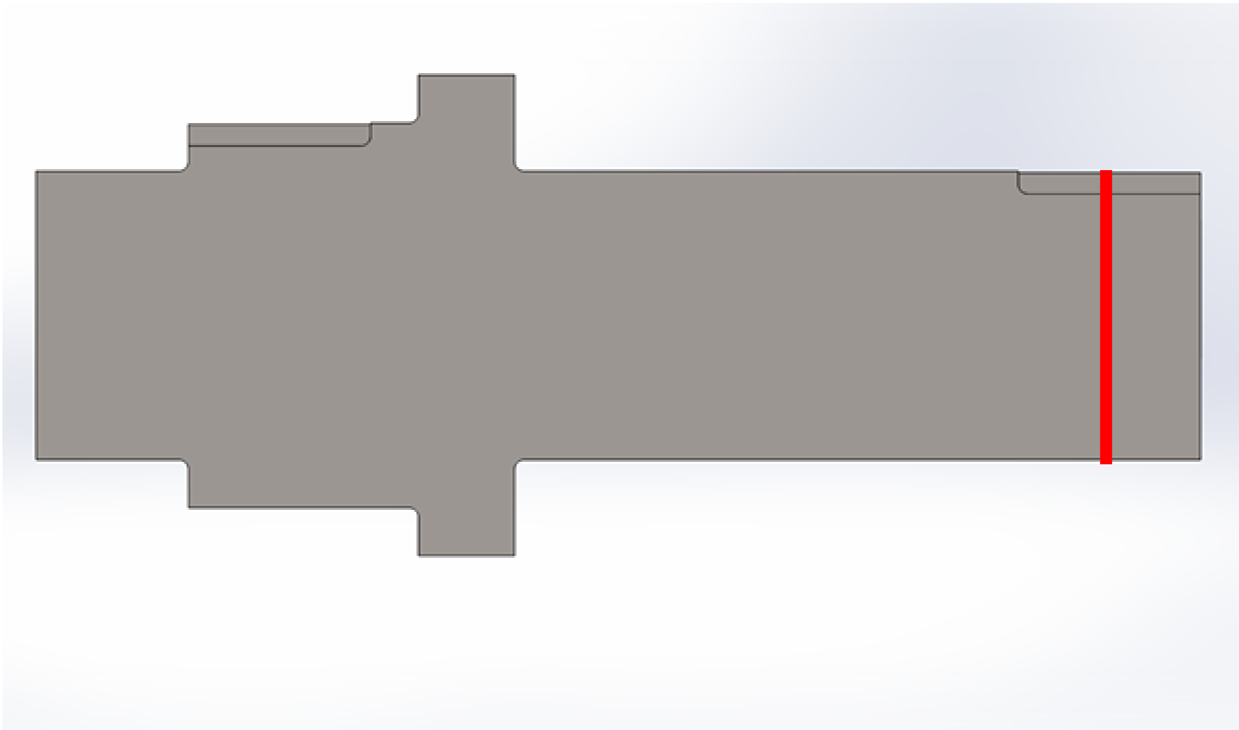


Figure 5: Image showing the critical area for the input shaft

For further analysis see Appendix B-2: Input Shaft Calculations.

3.2.2 Intermediate Shaft Fatigue Analysis

The portion of the intermediate shaft with the lowest factor of safety was found to be the left keyway, where the shaft interfaces with the input pinion. The safety factor on this portion was found to be 3.25, which indicates that this part is overbuilt for the task. This is acceptable because the cost of material is relatively low compared to the cost of machining custom parts. By using a shaft that has the same inner race for bearings and gears, we reduce the complexity of the assembly process, and can buy multiples of the same part.

This answer was found by following the same steps as the input shaft to calculate applied and reaction forces, resulting in the values that are tabulated in Table 6. Followed by the stresses at each of the major points on the shaft as seen by Table 7 and the safety factor for each of the chosen locations as seen by Table 8 with a detailed derivation in Appendix

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Input Shaft							Forces from FBD	Name	Distance	Force	x	y	z	
2									B1G	95	Fg1	0.00	0.00	10886.00	
3	Input Torque (Nm)	1632.9		Bearing Weight (N):	58.86				B1B2	135	Fg1a	2916.89	0.00	0.00	
4	Large Gear Diameter (mm)	300		Weight Gear	135.152				B1s	0	Fg1r	0.00	2916.89	0.00	
5	Input Speed (rpm)	37.8									Fws	0.00	0.00	0.00	
6	Gear Ratio	12.7									Frb2	0.00	5293.62	7565.41	
7	Shoulder height	10									Frb1	0.00	-2376.73	3185.44	
8	Shaft density	7850		Input Shaft Weight											
9	Gear density	7850		Section	1	2		3	4	5					
10	g (m/s^2)	9.81				gear									
11	Helix Angle	15			bearing				bearing	turbine					
12	Pressure Angle	15		Side view											
13	Sut	1.77E+09													
14	Sy	1.64E+09													
15	Preload	4956.23		Feature	shoulder	keyway	shoulder		shoulder	Input Keyway					
16				Diameter (mm)	35	45	45	55	35	35					
17				Length (mm)	25.4	25.4	0	10	25.4	25.4					
18	Points to consider			Cutouts (mm^2)		63		169.65		40					
19	1-2 shoulder			Cutouts (mm^3)		1280.16		169.65		812.8					
20	2 keyway			I of section (mm^4)	73662	177002	201289	449180	73662	65118					
21	2-3 shoulder			Volume (mm^3)	24438	39117	0	23589	24438	23625					
22	3-4 large shoulder			External vertical forces (N) (assumed applied	13374	-22971	0	0	9600	0					
23	5 keyway			Weight + forces (N)	13375.60	-22967.99	0.00	1.82	9601.5	1.82					
24				Moments - Equivalent weight moments at ce	-1	0	0	0	0	0					
25				Moments - Equivalent weight moments at Ed	-234	-218	-218	-218	122	0					
26				Total Weight (N)		12.7									
27															
28															
29	Input Shaft			DShaft1	Axial	Bending		DShaft2	Axial	Bending		DShaft3	Axial	Bending	
30	Ft (N)	21772.0		Cor. Se:	2.64E+08	3.77E+08		Cor. Se:	2.58E+08	3.68E+08		Cor. Se:	2.53E+08	3.61E+08	
31	Fa	7924.4		Cload	0.70	1.00		Cload	0.70	1.00		Cload	0.70	1.00	
32	Fr	7924.4		Csize	0.84	0.84		Csize	0.82	0.82		Csize	0.81	0.81	
33	Fw			Csurf	0.62	0.62		Csurf	0.62	0.62		Csurf	0.62	0.62	
34	Fpre	0		Ctemp	1.00	1.00		Ctemp	1.00	1.00		Ctemp	1.00	1.00	
35				Creliab	0.81	0.81		Creliab	0.81	0.81		Creliab	0.81	0.81	
36				Se'	8.86E+08	8.86E+08		Se	8.86E+08	8.86E+08		Se	8.86E+08	8.86E+08	

Figure 9. Input shaft data

38												
39	Se	3.68E+08	Point 1: Bearing to Shoulder						σ Bending nom	-5.56E+07	σ Bending Cor	-9.71E+07
40	Cor. Mean	3.20E+08	Kt Axial	2.08		Tension	0.00E+00		σ Axial nom	0.00E+00	σ Axial Cor.	0.00E+00
41	Cor. Amp	9.71E+07	Kt Moment	1.81		Torsion	1.63E+03		τ Torsion Nom	1.94E+08	τ Torsion Cor.	3.20E+08
42	OZ	3.34E+08	kt Torsional	1.65		Shear	0.00E+00					
43	ZS	2.00E+08	D/d:	1.29		Moment	-2.34E+02			σ Mean Vo	3.20E+08	
44	Cor. Mean @S	3.60E+08	r/d:	0.06		I (m^4)	7.37E-08			σ Alternati	9.71E+07	
45	Cor. Amp@S	2.93E+08	r:	2.00	<-- different	J	1.47E-07					
46	Safety Factor	1.60	q:	0.92		Area (m^2)	9.62E-04					
47			Kf Axial:	2.08	<- Kf = Kt							
48			Kf Moment:	1.75		shoulder						
49			Kf Torsional:	1.65	<- Kf = Kt							
50												
51												
52	Se	3.68E+08	Point 2: keyway						σ Bending nom	-2.77E+07	σ Bending Cor	-5.68E+07
53	Cor. Mean	2.72E+08	Kt Axial	1.00	assumed	Tension	0.00E+00		σ Axial nom	0.00E+00	σ Axial Cor.	0.00E+00
54	Cor. Amp	5.68E+07	Kt Moment	2.14	p.607	Torsion	1.63E+03		τ Torsion Nom	1.04E+08	τ Torsion Cor.	2.72E+08
55	OZ	2.78E+08	kt Torsional	2.62	p.607	Shear						
56	ZS	2.50E+08	D/d:	1.29		Moment	-2.18E+02			σ Mean Vo	2.72E+08	
57	Cor. Mean @S	3.23E+08	r/d:	0.02		I (m^4)	1.77E-07			σ Alternati	5.68E+07	
58	Cor. Amp@S	3.01E+08	r:	1.00		J	3.54E-07					
59	Safety Factor	1.90	q:	0.92		Area (m^2)	1.59E-03					
60			Kf Axial:	1.00	<- Kf = Kt							
61			Kf Moment:	2.05								
62			Kf Torsional:	2.62	<- Kf = Kt							
63												
64	Se	3.68E+08	Point 3: Gear to Shoulder						σ Bending nom	-1.69E+04	σ Bending Cor	-3.60E+04
65	Cor. Mean	1.98E+08	Kt Axial	2.54		Tension	7.92E+03		σ Axial nom	4.98E+06	σ Axial Cor.	1.27E+07
66	Cor. Amp	3.60E+04	Kt Moment	2.23		Torsion	1.63E+03		τ Torsion Nom	1.04E+08	τ Torsion Cor.	1.97E+08
67	OZ	1.98E+08	kt Torsional	1.90		Shear						
68	ZS	3.20E+08	D/d:	1.22		Moment	-1.33E-01			σ Mean Vo	1.98E+08	
69	Cor. Mean @S	2.63E+08	r/d:	0.02		I (m^4)	1.77E-07			σ Alternati	3.60E+04	
70	Cor. Amp@S	3.14E+08	r:	1.00		J	3.54E-07					
71	Safety Factor	2.62	q:	0.92		Area (m^2)	1.59E-03					
72			Kf Axial:	2.54	<- Kf = Kt							
73			Kf Moment:	2.13								
74			Kf Torsional:	1.90	<- Kf = Kt							

Figure 10. Input shaft safety factor calculations 1

75											
76	Se	3.68E+08	Point 4: Large Shoulder to Bearing					σ Bending nom	3.72E+07	σ Bending Cor	7.51E+07
77	Cor. Mean	3.75E+08	Kt Axial	2.49		Tension	7.92E+03	σ Axial nom	4.98E+06	σ Axial Cor.	1.24E+07
78	Cor. Amp	7.51E+07	Kt Moment	2.11		Torsion	1.63E+03	τ Torsion Nom	1.94E+08	τ Torsion Cor.	3.75E+08
79	OZ	3.83E+08	kt Torsional	1.93		Shear					
80	ZS	2.11E+08	D/d:	1.29		Moment	1.22E+02		σ Mean Vc	3.75E+08	
81	Cor. Mean @S	4.18E+08	r/d:	0.03		I (m^4)	7.37E-08		σ Alternati	7.51E+07	
82	Cor. Amp@S	2.81E+08	r:	1.00		J	1.47E-07				
83	Safety Factor	1.55	q:	0.92		Area (m^2)	1.59E-03				
84			Kf Axial:	2.49	<- Kf = Kt						
85			Kf Moment:	2.02							
86			Kf Torsional:	1.93	<- Kf = Kt						
87											
88	Se	3.68E+08	Point 5: Coupling to turbine (keyway)					σ Bending nom	6.21E+03	σ Bending Cor	1.16E+04
89	Cor. Mean	6.14E+08	Kt Axial	1.00	assumed	Tension	0.00E+00	σ Axial nom	0.00E+00	σ Axial Cor.	0.00E+00
90	Cor. Amp	1.16E+04	Kt Moment	1.95	p.607	Torsion	1.63E+03	τ Torsion Nom	2.19E+08	τ Torsion Cor.	6.14E+08
91	OZ	6.14E+08	kt Torsional	2.80	p.607	Shear					
92	ZS	2.36E+08	D/d:	1.29		Moment	2.31E-02		σ Mean Vc	6.14E+08	
93	Cor. Mean @S	6.62E+08	r/d:	0.03		I (m^4)	6.51E-08		σ Alternati	1.16E+04	
94	Cor. Amp@S	2.31E+08	r:	1.00		J	1.30E-07				
95	Safety Factor	1.38	q:	0.92		Area (m^2)	9.62E-04				
96			Kf Axial:	1.00	<- Kf = Kt						
97			Kf Moment:	1.87							
98			Kf Torsional:	2.80	<- Kf = Kt						

Figure 11. Input shaft safety factor calculations 2

B-3: Intermediate Shaft Calculations. Note that where the stress on the shaft is constant, such as the axial and torsional loading, $K_f = K_t$, so $q = 1$ in those cases.

Table 6: Forces from intermediate shaft FBD

Force	x	y	z
Reaction B1	0.00	2477.1	8008.0
Reaction B2	0.00	1258.3	-15.6
Gear 1	2916.89	2916.9	10886.0
Gear 2	818.50	818.5	3054.7
Weight G1	0.00	0.0	7.8
Weight G2	0.00	0.0	135.2
Weight Shaft	0.00	0	18.11

Table 7. Intermediate shaft fatigue data at left keyway

	Nominal Stress	K_t	q	K_f	Cor. Stress
Bending	2.61E+07	2.14	0.50	1.57	4.09E+07
Axial	0.00E+00	1.0	1	1.0	0.00E+00
Torsion	2.75E+07	2.62	1	2.62	7.20E+07

Table 8. Intermediate shaft Von Mises stress and safety factor

Mean Von Mises Stress	1.25E+08
Alternating Von Mises Stress	4.09E+07
Line OZ length	1.31E+08
Line ZS length	2.95E+08
Safety Factor	**3.25**

Figure 6: Image showing the critical area on the intermediate shaft

For further analysis see appendix section

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Input Shaft							Forces from FBD	Name	Distance	Force	x	y	z	
2									B1G	95	Fg1	0.00	0.00	10886.00	
3	Input Torque (Nm)	1632.9		Bearing Weight (N):	58.86				B1B2	135	Fg1a	2916.89	0.00	0.00	
4	Large Gear Diameter (mm)	300		Weight Gear	135.152				B1s	0	Fg1r	0.00	2916.89	0.00	
5	Input Speed (rpm)	37.8									Fws	0.00	0.00	0.00	
6	Gear Ratio	12.7									Frb2	0.00	5293.62	7565.41	
7	Shoulder height	10									Frb1	0.00	-2376.73	3185.44	
8	Shaft density	7850		Input Shaft Weight											
9	Gear density	7850		Section	1	2		3	4	5					
10	g (m/s^2)	9.81				gear									
11	Helix Angle	15			bearing				bearing	turbine					
12	Pressure Angle	15		Side view											
13	Sut	1.77E+09													
14	Sy	1.64E+09													
15	Preload	4956.23		Feature	*shoulder*	*keyway*	*shoulder*		*shoulder*	*Input Keyway*					
16				Diameter (mm)	35	45	45	55	35	35					
17				Length (mm)	25.4	25.4	0	10	25.4	25.4					
18	Points to consider			Cutouts (mm^2)		63		169.65		40					
19	1-2 shoulder			Cutouts (mm^3)		1280.16		169.65		812.8					
20	2 keyway			I of section (mm^4)	73662	177002	201289	449180	73662	65118					
21	2-3 shoulder			Volume (mm^3)	24438	39117	0	23589	24438	23625					
22	3-4 large shoulder			External vertical forces (N) (assumed applied	13374	-22971	0	0	9600	0					
23	5 keyway			Weight + forces (N)	13375.60	-22967.99	0.00	1.82	9601.5	1.82					
24				Moments - Equivalent weight moments at ce	-1	0	0	0	0	0					
25				Moments - Equivalent weight moments at Ec	-234	-218	-218	-218	122	0					
26				Total Weight (N)		12.7									
27															
28															
29	Input Shaft			DShaft1	Axial	Bending		DShaft2	Axial	Bending		DShaft3	Axial	Bending	
30	Ft (N)	21772.0		Cor. Se:	2.64E+08	3.77E+08		Cor. Se:	2.58E+08	3.68E+08		Cor. Se:	2.53E+08	3.61E+08	
31	Fa	7924.4		Cload	0.70	1.00		Cload	0.70	1.00		Cload	0.70	1.00	
32	Fr	7924.4		Csize	0.84	0.84		Csize	0.82	0.82		Csize	0.81	0.81	
33	Fw			Csurf	0.62	0.62		Csurf	0.62	0.62		Csurf	0.62	0.62	
34	Fpre	0		Ctemp	1.00	1.00		Ctemp	1.00	1.00		Ctemp	1.00	1.00	
35				Creliab	0.81	0.81		Creliab	0.81	0.81		Creliab	0.81	0.81	
36				Se'	8.86E+08	8.86E+08		Se	8.86E+08	8.86E+08		Se	8.86E+08	8.86E+08	

Figure 9. Input shaft data

38											
39	Se	3.68E+08	Point 1: Bearing to Shoulder					σ Bending nom	-5.56E+07	σ Bending Cor	-9.71E+07
40	Cor. Mean	3.20E+08	Kt Axial	2.08		Tension	0.00E+00	σ Axial nom	0.00E+00	σ Axial Cor.	0.00E+00
41	Cor. Amp	9.71E+07	Kt Moment	1.81		Torsion	1.63E+03	τ Torsion Nom	1.94E+08	τ Torsion Cor.	3.20E+08
42	OZ	3.34E+08	kt Torsional	1.65		Shear	0.00E+00				
43	ZS	2.00E+08	D/d:	1.29		Moment	-2.34E+02		σ Mean Vo	3.20E+08	
44	Cor. Mean @S	3.60E+08	r/d:	0.06		I (m^4)	7.37E-08		σ Alternati	9.71E+07	
45	Cor. Amp@S	2.93E+08	r:	2.00	<-- different	J	1.47E-07				
46	Safety Factor	1.60	q:	0.92		Area (m^2)	9.62E-04				
47			Kf Axial:	2.08	*<- Kf = Kt*						
48			Kf Moment:	1.75		*shoulder*					
49			Kf Torsional:	1.65	*<- Kf = Kt*						
50											
51											
52	Se	3.68E+08	Point 2: keyway					σ Bending nom	-2.77E+07	σ Bending Cor	-5.68E+07
53	Cor. Mean	2.72E+08	Kt Axial	1.00	*assumed*	Tension	0.00E+00	σ Axial nom	0.00E+00	σ Axial Cor.	0.00E+00
54	Cor. Amp	5.68E+07	Kt Moment	2.14	*p.607*	Torsion	1.63E+03	τ Torsion Nom	1.04E+08	τ Torsion Cor.	2.72E+08
55	OZ	2.78E+08	kt Torsional	2.62	*p.607*	Shear					
56	ZS	2.50E+08	D/d:	1.29		Moment	-2.18E+02		σ Mean Vo	2.72E+08	
57	Cor. Mean @S	3.23E+08	r/d:	0.02		I (m^4)	1.77E-07		σ Alternati	5.68E+07	
58	Cor. Amp@S	3.01E+08	r:	1.00		J	3.54E-07				
59	Safety Factor	1.90	q:	0.92		Area (m^2)	1.59E-03				
60			Kf Axial:	1.00	*<- Kf = Kt*						
61			Kf Moment:	2.05							
62			Kf Torsional:	2.62	*<- Kf = Kt*						
63											
64	Se	3.68E+08	Point 3: Gear to Shoulder					σ Bending nom	-1.69E+04	σ Bending Cor	-3.60E+04
65	Cor. Mean	1.98E+08	Kt Axial	2.54		Tension	7.92E+03	σ Axial nom	4.98E+06	σ Axial Cor.	1.27E+07
66	Cor. Amp	3.60E+04	Kt Moment	2.23		Torsion	1.63E+03	τ Torsion Nom	1.04E+08	τ Torsion Cor.	1.97E+08
67	OZ	1.98E+08	kt Torsional	1.90		Shear					
68	ZS	3.20E+08	D/d:	1.22		Moment	-1.33E-01		σ Mean Vo	1.98E+08	
69	Cor. Mean @S	2.63E+08	r/d:	0.02		I (m^4)	1.77E-07		σ Alternati	3.60E+04	
70	Cor. Amp@S	3.14E+08	r:	1.00		J	3.54E-07				
71	Safety Factor	2.62	q:	0.92		Area (m^2)	1.59E-03				
72			Kf Axial:	2.54	*<- Kf = Kt*						
73			Kf Moment:	2.13							
74			Kf Torsional:	1.90	*<- Kf = Kt*						

Figure 10. Input shaft safety factor calculations 1

75											
76	Se	3.68E+08	Point 4: Large Shoulder to Bearing					σ Bending nom	3.72E+07	σ Bending Cor	7.51E+07
77	Cor. Mean	3.75E+08	Kt Axial	2.49		Tension	7.92E+03	σ Axial nom	4.98E+06	σ Axial Cor.	1.24E+07
78	Cor. Amp	7.51E+07	Kt Moment	2.11		Torsion	1.63E+03	τ Torsion Nom	1.94E+08	τ Torsion Cor.	3.75E+08
79	OZ	3.83E+08	kt Torsional	1.93		Shear					
80	ZS	2.11E+08	D/d:	1.29		Moment	1.22E+02		σ Mean Vc	3.75E+08	
81	Cor. Mean @S	4.18E+08	r/d:	0.03		I (m^4)	7.37E-08		σ Alternati	7.51E+07	
82	Cor. Amp@S	2.81E+08	r:	1.00		J	1.47E-07				
83	Safety Factor	1.55	q:	0.92		Area (m^2)	1.59E-03				
84			Kf Axial:	2.49	<- Kf = Kt						
85			Kf Moment:	2.02							
86			Kf Torsional:	1.93	<- Kf = Kt						
87											
88	Se	3.68E+08	Point 5: Coupling to turbine (keyway)					σ Bending nom	6.21E+03	σ Bending Cor	1.16E+04
89	Cor. Mean	6.14E+08	Kt Axial	1.00	assumed	Tension	0.00E+00	σ Axial nom	0.00E+00	σ Axial Cor.	0.00E+00
90	Cor. Amp	1.16E+04	Kt Moment	1.95	p.607	Torsion	1.63E+03	τ Torsion Nom	2.19E+08	τ Torsion Cor.	6.14E+08
91	OZ	6.14E+08	kt Torsional	2.80	p.607	Shear					
92	ZS	2.36E+08	D/d:	1.29		Moment	2.31E-02		σ Mean Vc	6.14E+08	
93	Cor. Mean @S	6.62E+08	r/d:	0.03		I (m^4)	6.51E-08		σ Alternati	1.16E+04	
94	Cor. Amp@S	2.31E+08	r:	1.00		J	1.30E-07				
95	Safety Factor	1.38	q:	0.92		Area (m^2)	9.62E-04				
96			Kf Axial:	1.00	<- Kf = Kt						
97			Kf Moment:	1.87							
98			Kf Torsional:	2.80	<- Kf = Kt						

Figure 11. Input shaft safety factor calculations 2

B-3: Intermediate Shaft Calculations.

3.3 Gear and Pinion Specifications

Due to a symmetric design of input and output shafts there is only one pinion and gear design. The loads on the output shaft are far lower, as such it should not be the limiting factor. The gears and pinions are made from Carburized and case hardened 64 HRC. The pitch radii are dependant on the shaft diameter and an appropriate value for the face width has been chosen based on the shaft analysis. The pressure and helix angles, and gear quantity have been chosen during the FBD creation. Please reference Appendix B-5: Gears and Pinions for sample calculations and data tables.

The first step in the analysis was to determine the number of teeth on the gear and pinions to be analysed. Based on the gear ratio m_g of 3.57 as well as taking into consideration the helix angle and pressure angles, the appropriate number of teeth for the gear and pinion are 125 and 35, respectively. Once the number of teeth has been determined the modulus m can be calculated using $m = \frac{2 * Pitch\ radius}{Number\ of\ gear\ teeth}$ (12.4c) [1]. The addendum is equivalent to the module and the dedendum is $modulus * 1.25$. Now then the length of action Z, can be determined using the following formula $Z = \sqrt{(r_p + a_p)^2 - (r_p \cos \emptyset)^2} + \sqrt{(r_g + a_g)^2 - (r_g \cos \emptyset)^2} - C \cos \emptyset$

(12.2) [1], where a) r_p, r_g are pitch circle radii b) a_p, a_g are the addenda and c) C is the center to center distance, equivalent to $r_p + r_g$. Lastly, the contact ratio is $m_p = \frac{Z}{m*\pi*\cos\emptyset}$(12.7b) [1].

Table 9: Gear and pinion specifications

Specifications for:	Number of Teeth	Module	Addendum	Dedendum	Contact ratio	Gear Quantity	Pitch Radius	Pressure and helix angle	Face width
Gear	125	2.4	2.4	3.0	5.708	12	150	15	25.4
Pinion	35		2.4	3.0		12	42.09		

3.3.1 Input Gear Fatigue Analysis

The lower safety factor was found to be 1.508 for bending failure. Which was expected to be the worst due to the high amount of forces at the input. It is assumed that a) the number of cycles this gear is to withstand is $4 * 10^4$ cycles with regards to the K_L value calculation b) the operating temperature will not exceed 250F and c) a reliability of 99% is good enough for this project. Due to the very high load this gear will require maintenance every 26h, which is unacceptable and therefore require further iteration.

The safety factor for bending is calculated by $N_b = \frac{S_{fb}}{\sigma_b}$. The bending-fatigue strength for the gear is $S_{fb} = \frac{K_L*S_{fb'}}{K_T*K_R}$(12.24a) [1], consisting of and uncorrected value $S_{fb'}$ from a table which is corrected based on the use of the gear. For the life factor K_L the assumed number of load cycles is $4 * 10^4$ which using the equation $K_L = 6.1514(4 * 10^4)^{-0.1192}$,Figure 12-24 [1] gives the value of 1.657 for K_L. Since T < 250 F, therefore the temperature factor K_T is 1. And the reliability factor K_R is 1 from table 12-19 [1]. Next the Bending stress will have to be calculated with $\sigma_b = \frac{W_t*K_a*K_m*K_s*K_B*K_I}{F*m*J*K_v}$(12.15si) [1]. The load distribution factor K_m can be taken from table 12-16 [1] with knowing the face width value of 25.4 mm. K_a is the application factor which from table 12-17 [1] is 1, uniform driving machine and uniform driven machine. the size factor K_s is 1 for this size gear. The rim thickness factor K_b is 1. Idler factor K_I is 1 for non-idler gears. The dynamic factor K_v is estimated using $K_v = (\frac{A}{A+\sqrt{200*V_t}})^B$(12.16si) [1] for metric units, where

$A = 50 + 56 * (1 - B)$(12.17b) [1] and $B = \frac{(12-Q_v)^{2/3}}{4}$(12.17b) [1]. F is the face width, m is the modulus and J can be found from the table on pg. 23 of [3].

The safety factor for surface failure is calculated by $N_C = \left(\frac{S_{fc}}{\sigma_c}\right)^2$. The surface-fatigue strength for the gear is $S_{fc} = \frac{C_L * S_{fb'} * C_H}{C_T * C_R}$(12.25) [1], consisting of and uncorrected value from a table which is corrected based on the use of the gear. For the life factor C_L the assumed number of load cycles is $4 * 10^4$ which using figure 12-26 [1] approximately gives the value of 1.332 for C_L. The hardness ratio factor $C_H$is 1 since both gear and pinion are of the same hardness. Since T < 250 F, therefore the temperature factor C_T is 1. And the reliability factor C_R is 1 from table 12-19 [1]. Next the Surface stress will have to be calculated with $\sigma_c = C_p * \sqrt{\frac{W_t * C_a * C_m * C_s * C_f}{F * I * d * C_v}}$(12.21) [1], where F is the face width and d is the pitch diameter of the smaller gear. The load distribution factor C_m can be taken from table 12-16 [1], with knowing the face width value of 25.4 mm. C_a is the application factor which from table 12-17 [1] is 1, uniform Driving machine and uniform driven machine. The size factor C_s is 1 for this size gear. Surface finish factor C_f is assumed to be 1 for gears made by conventional ways [1]. The dynamic factor C_v is estimated using $C_v = \left(\frac{A}{A + \sqrt{200 * V_t}}\right)^B$ for metric units, where $A = 50 + 56 * (1 - B)$ and $B = \frac{(12-Q_v)^{2/3}}{4}$. The last coefficient needed is the elastic coefficient C_p, which can be found using the equation $C_p = \sqrt{\frac{1}{\pi * [\left(\frac{1-v_p^2}{E_p}\right) + \left(\frac{1-v_g^2}{E_g}\right)]}}$(12.23) [1], where v and E are the poison ratio and moduli of elasticity for the gears and pinions. Lastly, I is the surface geometry factor which can be calculated from $I = \frac{\cos \emptyset}{\left(\frac{1}{\rho_p} + \frac{1}{\rho_g}\right) * d_p * m_n}$(13.6a) [1]. To find I the following variables need to be calculated:

1. Radius of curvature (ρ_p, ρ_g)
2. Normal pressure angle ($\emptyset_n$)
3. Base helix angle (φ_b)
4. Axial contact ratio (m_F)

5. Transverse contact ratio (m_p)
6. Minimum length of the lines of contact (L_{min})
7. Load-sharing ratio (m_n)

The radius of curvature ρ_p, ρ_g needs to be calculated for each the pinions and gears using $\rho_p = \sqrt{\{0.5[(r_p + a_p) + (C - r_g - a_g)]\}^2 - (r_p * \cos \emptyset)^{\wedge}2}$ (13.6g) [1] and $\rho_g = C * \sin \emptyset - \rho_p$(13.6g) [1] ,respectively. The second variable needed is the normal pressure angle $\emptyset_n$ which depends on the helix and pressure angles, $\emptyset_n = \tan^{-1}(\cos \varphi * \tan \emptyset)$, rearranged equation (13.2) [1]. The third variable required is the base helix angle $\varphi_b = \cos^{-1}(\cos(\varphi) * \frac{\cos \emptyset_n}{\cos \emptyset})$ (13.6f) [1]. The fourth and fifth variables are the axial contact ratio, $m_F = \frac{F*\tan \varphi}{m*\pi}$(13.5) [1] and the transverse contact ratio (another name for the contact ratio), from which only the residuals are needed, $n_a \ and \ n_r$. Now since $n_a > 1 - n_r$ the minimum length of the lines of contact is $L_{min} = \frac{m_p*F-(1-n_a)*(1-n_r)*p_x}{\cos \varphi_b}$(13.6e) [1]. Lastly, the seventh and final variable is the load-sharing ratio $m_n = \frac{F}{L_{min}}$ (13.6b) [1] and after it is calculated, I can finally be determined, along with the safety factor.

Table 10: Input gear fatigue data

	Stress (MPa)	Uncorrected Bending Strength (MPa)	Corrected Bending Strength (MPa)	Safety Factor
Bending	571	520	861.8	1.508
Surface	712.66	1300	1731	5.901

3.3.2 Intermediate Pinion Fatigue Analysis

The lower safety factor for this pinion was calculated to be 2.873 for surface failure. It is assumed that a) the number of cycles this gear is to withstand is 10^{10} cycles b) the operating temperature will not exceed 250F and c) a reliability of 99% is good enough for this project. The estimated service interval for this pinion is every $1.24 * 10^6$ hours.

Table 11: Intermediate pinion fatigue data

	Stress (MPa)	Uncorrected Bending Strength (MPa)	Corrected Bending Strength (MPa)	Safety Factor
Bending	162.9	520	467.9	2.873
Surface	195.4	1300	883	20.42

3.3.3 Intermediate Gear Fatigue Analysis

The lower safety factor was found to be 2.918 for surface failure. A higher safety factor is to be expected since there is less loading. It is assumed that a) the number of cycles this gear is to withstand is 10^{10} cycles with regards to the K_L value calculation b) the operating temperature will not exceed 250F and c) a reliability of 99% is good enough for this project. The estimated service interval for this gear is every $1.24 * 10^6$ hours.

Table 12: Intermediate gear fatigue data

	Stress(MPa)	Uncorrected Bending Strength (MPa)	Corrected Bending Strength (MPa)	Safety Factor
Bending	160.4	520	467.9	2.918
Surface	377.5	1300	883	5.470

3.3.4 Key Failure

Generally, keys are designed to act like mechanical fuses for if an unexpected change in loading is introduced, and a softer material is part of this design. In this case, there was an error in our initial safety factor calculations, and our lowest acceptable safety factor in the shafts dropped to 1.38, so our key is no longer weaker than the shaft. Since we did not want to lower the safety factor further, we chose to use the same material that is used on the shafts for simplicity, 4140 Steel Q&T at 400°F which has a yield strength of 1.64 GPa. For coupling the gears and pinions to their respective shafts, the parallel key used has slightly different dimensions than the one that couples the input and output shafts to the turbine and generator. The key used for the gears and pinions has a width of 14 mm, a height of 9mm, a length of 25.4 mm, and the underlying keyway has inside corner radii of 1 mm. The input/output shaft key has a width of 10 mm, a height of 8mm, a length of 25.4 mm, with the same inside corner radii of the keyway. These dimensions were partly determined by the standard sizes listed in Table 10-2 of Norton [1]. For this analysis, we will assume that the key is placed with half of its height in the coupled part, and half in the base shaft.

A key's failure mode is either in shear or in bearing failure. The input/output key that would have the highest shear probability is the initial coupling to the input shaft, where it has the highest torque. For the gear/pinion key, the highest shear potential is in the first pinion, found on the intermediate shaft. The formula for shear is found in (10.10) [1], where *F* is the torque divided by the radius of the shaft, and A_{shear} is the cross-sectional area of the key that is being sheared. The formula for bearing stress is shown in (10.11) [1], where *F* is the same as for (10.10) [1], and $A_{bearing}$ is the area of contact between the key and the shaft. The numerical results of these calculations are tabulated in Table 13.

$$(10.10)\ \tau = \frac{F}{A_{shear}}$$

$$(10.11)\ \sigma = \frac{F}{A_{bearing}}$$

Table 13. Key failure calculation data

Key	F (N)	A_{shear} (m^2)	$A_{bearing}$ (m^2)	τ (Pa)	σ (Pa)
Input key	9.33E+04	2.54E-04	1.02E-04	3.67E+08	9.18E+08
First pinion key	2.04E+04	3.56E-04	1.14E-04	5.73E+07	1.78E+08

The safety factor for shear and bearing failure can be calculated by the following equations from Norton [1], and the resultant safety factors are given in Table 14, where S_y is 1.19 GPa. The lowest resultant safety factor is 1.79, from the input key in bearing failure. A sample calculation is given in Appendix B-4: Key Failure Calculations.

$$(10.10)\ SF_{shear} = \frac{0.5 * S_y}{\tau}$$

$$SF_{bearing} = \frac{S_y}{\sigma}$$

Table 14. Safety factor for key shear and bearing failure

	Input key	First pinion key
SF_{shear}	2.23	14.33
$SF_{bearing}$	**1.79**	9.21

3.4 Bearing Analysis

In order to complete the bearing analysis, it is first necessary to calculate all of the reaction forces in each of the 6 bearings this was completed previously for the input and secondary shafts and the same process was used to calculate the reactions in the output shaft and the results are seen in Table 15. The bearings are numbered from 1 starting at the outer input bearing and 6 being outer output bearing. The preload for the bearings is then calculated by examining the axial force generated by the angle of the bearing race and radial bearing forces then subtracting the axial force from the gears as per the SKF bearing preload formula and then the preload is set to be the higher value in each bearing pair [4]. The results of the calculations can be seen in Table 15. After calculating the preload required for the bearing pairs the bearings were analysed to find the expected life with 99% reliability. It was then necessary to examine the proportion of Axial and Radial bearing force to verify the constant applied load P, for the bearing selected this was an SKF 32207 which has a threshold of 0.37 Appendix C-18 Figure 48: Bearing datasheet. Under this value the constant applied load is defined as the radial load and above is defined as 0.4(Fr)+1.6(Fa). This process is outlined in Norton 11.10 [1] and the formula used to calculate life span was 11.20d [1] which is represented as L_P (10^6 cycles) which was then converted into Service Interval based on a 100% duty cycle [1]. The results of this calculation can be viewed in Table 15 and a sample calculation for bearing # 3 can be found in Appendix B-6: Bearings Calculations.

$$Internal\ Axial\ Bearing\ Force = \frac{F_r}{Y(defined\ by\ bearing)}$$

$$Preload = |Fa - Internal\ Axial\ Bearing\ Force|$$

$$(11.20d) \quad L_P = K_R \left(\frac{C}{P}\right)^{10/3}$$

$$Service\ interval = \frac{L_P(10^6)}{60(RPM\ of\ shaft)}$$

Bearing summaries	Fr (N)	Fa (N)	Internal axial force (N)	Required Preload (N)	Fa/VFr (N)	P (N)	L10 (million cycles)	Lp (million cycles)	S i (
1	7929.961963	0	4956.23	4956.23	0	7929.962	2331.39799	489.5936	2
2	9233.52106	2916.89	5770.95	2854.06	0.315903	9233.521	1403.7689	294.7915	1
3	8603.974632	2916.89	5377.48	2460.59	0.339017	8603.975	1776.32745	373.0288	4
4	1247.150857	818.50	779.47	39.03	0.656296	1808.461	321728.517	67562.99	8
5	1040.65622	372.7989	650.41	277.61	6.77E-05	1040.656	2029998.96	426299.8	1
6	421.7008594	0	263.56	263.56	0	421.7009	41226058.1	8657472	3

Table 15: Bearing analysis

3.5 Gearbox Housing

The gearbox housing will be cast from aluminum and will weigh approximately 36.5 kg empty, or 68 kg with the completed assembly inside. The seats for the bearings, end cap holes, and all fastener holes will be machined after the casting process, to ensure a smooth fit. The fastener holes for attaching the top and side endcaps will then be tapped, while the side pieces that will allow the housing to be fastened to the bedplate will not be. The side endcaps and top end cap will be cast out of aluminum also. Through holes will be drilled in each cap for the fasteners to attach through. No significant stress analysis was performed on the housing.

3.6 Fastener Analysis

Bolts are used to locate the entire assembly to the bedplate and machine screws are mated to the housing with tapped holes. In the following sections, the various end caps are analyzed for failure conditions, and a suitable preload is determined. The lowest safety factor in this section is 1.13, which is normal for a 90% preload. Note that due to design choice there will be no shear in any of the end caps and that the top cap is going to experience no further load beyond the preload. Please reference Appendix B-7 for sample calculations and data tables.

3.6.1 Input and Output Endcaps

Both the input and output end caps will use 4 machine screws to locate the shafts in place. The type will have the designation M5-0.8, will be made of steel, and will have an SAE grade of 5.8. The yielding safety factor for the bolts is calculated to be approximately 1.16 and the separation safety factor is 2.26.

The procedure to calculate the safety factor starts off with determining the total bolt length, l_{bolt}. The threaded length $l_{thd} = 2 * d + 6$, which is used to calculate the shank length $l_s = l_{bolt} - l_{thd}$. Further, the shank length can be used to find the length of thread in the clamp zone $l_t = l - l_s$. Moving on to calculating forces and force loads. First, to calculate the pre-load equation 15-1a [1] is used $F_i = 0.9 * S_p * A_t$, and then determining bolt stiffness, $k_{b'} = \left(1 + \frac{d}{l}\right)^{-1} \frac{A_t * A_b}{A_b * l_t + A_t * l_s} E_b$(15.17) [1]. Then the plate to bolt modulus ratio $r = \frac{E_{material}}{E_{bol}}$ (15.18b) [1] and joint aspect ratio $j = \frac{d}{l}$ (15.18d) [1] are calculated so that the joint factor can be determined $C = C_r = p_3 * r^3 + p_2 * r^2 + p_1 * r + p_0$(15.19) [1], where the p_i are given in Table 15-8 [1]. The factor can be used to calculate:

a) the material stiffness from the relationship $C = \frac{k_{b'}}{k_{b'} + k_m}$(15.13c) [1] which gives $k_m = k_{b'}(\frac{1-C}{C})$ [1]

b) the portions of the applied load felt by the bolt $P_b = C * P$(15.13c) [1]

c) the material load $P_m = (1 - C) * P$(15.13d) [1].

After, the resulting load can be found on the machine screw and material, $F_b = F_i + P_b$(15.14b) [1] and $F_m = F_i - P_m$(15.14a) [1] respectively. The last load that needs to be calculated is the load P_0 which is the minimum required to separate the joint, using the following equation $P_0 = \frac{F_i}{(1-C)}$ (15.14c) [1]. The stress in the bolt is $\sigma_b = \frac{F_b}{A_t}$, which allows the safety factor against yielding to be determined $N_y = \frac{S_y}{\sigma_b}$ and the safety factor against joint separation to be $N_{separation} = \frac{P_0}{P}$.

3.6.2 Intermediate Endcaps

The input side intermediate end cap will use 5 M6-1 bolts and the output side will use 4 M5-0.8 to locate each shaft in place. Each bolt will be made with a SAE grade of 5.8. The safety factor against yielding for the bolts is calculated to be approximately 1.210 and 1.228, and against separation to be 17.80 and 632.8, respectively. The yield safety factors are consistent with our applied preload of 90%. The machine screw calculations are the same as the ones done for the input and output shaft endcaps.

3.6.3 Gearbox Housing to Ground

The gear box housing was designed to be fastened to the bedplate at each of the four corners. To calculate the forces on the fasteners it was necessary to make two simplifying assumptions. The first is to assume that the gearbox can be treated as closed system with the input and output torque acting on the axis passing through the center of the shafts. The second is to recognize that the reactions differ little along the shaft axes, and therefore the fasteners on each short side of the gearbox can be treated as bolt pairs, sharing the load equally. Under these assumptions it was found that the force on the input/output side of case is 2575N and the 3240N on the other. The forces will be transferred through two class 8.8 M8 x 30mm bolts each to a bedplate. Then the bolts were analysed for static failure through the same method outlined in chapter 15 of Norton [1]. This method involved examining the stiffness of the bolt (kb) against and the material (km) to determine the proportion of the load on then felt by the bolt (Pb) then determining the maximum stress in the bolt (σ_b). By comparing the Yield strength to the stress felt by the bolt the minimum safety factor was found to be $N_y = 1.13$. Finally, by comparing the load required to separate the joint to the applied load the safety factor for separation was found to be $N_{Separtion} = 1.042$. Complete calculations can be found in Appendix B-7: Fastener Calculations.

4.0 Stakeholders

The only stakeholder for the land for the micro hydro project would be the City of Nanaimo, as the selected location resides in Nanaimo Zone H. The nearest First Nations band is located approximately 50 km away in Qualicum Bay, and they do not have claim to this land. For this project it is assumed that legal access to the land will be acquired from the city of Nanaimo, however there will be a need to remove some trees to bring in the equipment, and install the micro hydro system, which will require the submission of an "Application for a License to Cut" through the Ministry of Forests [5]. It is worth noting that the addition of a micro hydro system to this area could be used to power a small building, which may attract private investors for a campsite, wilderness retreat, or fishing hut [5]. A final consideration for the project is that the river supports a moderate fish population [6] and will require a grate to protect them from harm and to prevent any undo maintenance.

5.0 Conclusion

We were tasked with designing a micro-hydro installation with the goal of creating electricity. The location we selected is in the Nile Creek, near Qualicum Bay, with a 20 m head. The land will need to be acquired from the City of Nanaimo, and an Application for a License to Cut will need to be submitted through the Ministry of Forests for any tree-falling required for the construction of the project. A grate will be installed on the intake to bar the local fish from entering the turbine. The turbine will have a specific speed of 60, a flow rate of 33 L/s, and a power output of 6.45 kW. The equivalent gearbox ratio chosen is 12.7:1. We designed the gearbox with two helical gearsets, which employs identical input and output shafts, a unique intermediate shaft, and an even gear ratio across both gearsets. The lowest safety factor of the system is on the input shaft, at a value of 1.38. For the intermediate shaft, the safety factor is 3.25. The output shaft is identical to the input shaft, but with less applied torque and therefore will not fail before the input shaft. The safety factor on the gearset is 1.515 from tooth bending failure, with a service interval of 26 hours. The bearings will have a service interval of 5.27 years. The lowest safety factor of the fasteners is 1.13, which is expected for a 90% preload. Currently, further iteration on the shaft size and the gear service interval is required, and reasonable tolerancing needs to be determined before the project is complete.

6.0 Recommendations

In designing the gear box, we iterated over the shaft sizes and other factors a few times until we reached acceptable values. Unfortunately, in the process of iteration, some values were not updated correctly. In accordance with the ANSI Standard, parallel keys are to be of certain dimensions for each size of shaft. While verifying results, it was noticed that the key and keyway dimensions needed to be adjusted to match the increased shaft sizes. The shaft cross sections decreased because of this, lowering the safety factor to 1.38. Future work will include iterating at least one more time to get the lowest safety factor into the range of 1.5 to 2. Another error that was uncovered while doing sample calculations is that the gears spreadsheet was referencing a much smaller torque than was being applied. This decreased the time between servicing to 26 hours – an unacceptably low value. To fix this error, the face width of all gears should be increased to about 45 mm, which would allow the number of cycles between service intervals to be increased to the more comfortable range of 10^7 to 10^{10}, and would allow the increase of the safety factor to be further above 1.5. It will also increase the length of all shafts, resulting in all the CAD drawings and bending calculations of the shafts needing to be redone. When planning the next steps, part tolerancing should be considered, as it was outside the scope of this project. To accomplish this, the thermal expansion of the shafts at the operating temperature of the gearbox will need to be determined. After these three tasks are complete and the results are verified, we are confident that this product will be ready to be manufactured, assembled, and installed at the project location.

7.0 References

[1] R. L. Norton, Machine Design, Upper Saddle River, New Jersey: Pearson Education, Inc., 2014.

[2] H. Bailey, "Project Description Version 4," University of Victoria, Victoria, 2018.

[3] American Gear Manufacturers Assocation, "AGMA 908-B89 Information Sheet - Geometry Factors for Determining the Pitting Resistance and Bending Strength of Spur, Helical and Herringbone," August 1989. [Online]. Available: https://eclass.upatras.gr/modules/document/file.php/MECH1178/18.%20%CE%9C%CE%B5%CF%84%CF%89%CF%80%CE%B9%CE%BA%CE%BF%CE%AF%20%CE%BF%CE%B4%CE%BF%CE%BD%CF%84%CF%89%CF%84%CE%BF%CE%AF%20%CF%84%CF%81%CE%BF%CF%87%CE%BF%CE%AF/AGMA-908-B89.pdf. [Accessed 15 November 2018].

[4] "SKF - Bearing Preload," [Online]. Available: http://www.skf.com/binary/12-299896/Bearing0preload_tcm_12-299896.pdf. [Accessed 15 November 2018].

[5] John Croockewit, "Handbook for Developing Micro Hydro In British Columbia," BC Hydro, 2004.

[6] A. Kolasinski, "Island Angler," [Online]. Available: http://www.islandangler.net/fishing%20reports.htm. [Accessed 15 November 2018].

Appendix A: Peer Review Sheet – Final Report

THE UNIVERSITY OF VICTORIA
DEPARTMENT OF MECHANICAL ENGINEERING

MECH 360 – DESIGN OF MECHANICAL ELEMENTS

PEER REVIEW SHEET – FINAL REPORT

Group Number: 3

Please indicate which group members were directly involved in the following tasks. (Initials / forenames)

Report generation	Bryce, everyone ← A lot
CAD drawings	Struan
Shaft calculations	Bryce, Struan, Derek
Gear calculations	Martin
Bearing calculations	Derek
Gearbox housing and bolt calculations	Derek and Martin

Based on collective agreement of each team member's performance, allocate from 0% to 100% for each team member. 100% for a group member means he/she will receive the full group mark, and anything less indicates that group member may receive a portion of the group mark. As an example, a group of Jimmy (95%), Jane (100%), John (100%) and Julio (100%) receives a collective group mark of 79%. Jane, John and Julio will both receive 79% for their project mark, but Jimmy may receive 95% x 79% = 75% for his project mark. Be aware of the implications of the marking structure that may be applied using your point allocation to group marks. However, the final choice of whether marks will be reduced will depend on the TA / Dr Bailey, this will be based on the reasons given in the table, tasks completed, any previous relevant discussions and any other relevant factors.

Consider each person's overall contribution (including work, communication, problem solving, etc.) and give due consideration to their academic ability. If you feel any member deserves less/more than an equal share, indicate your reason(s).

Each group member must sign this form to indicate a collective group agreement. This signature refers to agreement for the team members' performance and for the tasks undertaken.

Member Name	Signature	%	Reason(s) if < 100%
Bryce Dombrowski		100	
Derek Smith		100	
Struan Eaves		100	
Martin Gospodinov	Mr	100	

Figure 7. Peer review sheet

Appendix B: Sample Calculations

B-1: Sample Shaft Calculation

Shaft Calculations

Stress Concentration Factor:

$$K_f = 1 + q(K_t - 1)$$

K_f (Bending @ big sholder Input shaft):

$K_t = 2.11 \quad q = 0.92$

$K_f = 1 + 0.92(2.11 - 1) = 2.0212$ ✓

$\sigma_a = 37.2\ MPa \qquad \sigma'_a = (K_{f\,Bending})\,\sigma_a = 75.14\ MPa$

Same process for Mean and Alternating for Bending, Axial and Torsional Stresses

$$\sigma'_m@S = \frac{S_{ut}(S_e^2 - S_e\sigma'_a - S_{ut}\sigma'_m)}{S_e^2 + S_{ut}^2}$$

$$= 4.18 \times 10^8\ Pa$$ ✓

$$\sigma'_a@S = -\frac{S_e}{S_{ut}}(\sigma'_m@S) + S_e$$

$$= -\frac{3.68\times10^8}{1.77\times10^9}(4.18\times10^8) + 3.68\times10^8 = 2.81\times10^8\ Pa$$ ✓

$$ZS = \sqrt{(\sigma'_m - \sigma'_m@S)^2 + (\sigma'_a - \sigma'_a@S)^2} = 2.11\times10^8$$ ✓

$$OZ = \sqrt{(\sigma'_a)^2 + (\sigma'_m)^2} = \sqrt{(75.14\,MPa)^2 + (3.75\times10^8)^2}$$

$$= 3.82\times10^8$$ ✓

$$N_f = \frac{OZ + ZS}{OZ} = \frac{3.82\times10^8 + 2.11\times10^8}{3.82\times10^8} = 1.55$$ ✓

Figure 8: Input shaft safety factor calculations for the large shoulder

B-2: Input Shaft Calculations

Input Shaft	
Input Torque (Nm)	1632.9
Large Gear Diameter (mm)	300
Input Speed (rpm)	37.8
Gear Ratio	12.7
Shoulder height	10
Shaft density	7850
Gear density	7850
g (m/s^2)	9.81
Helix Angle	15
Pressure Angle	15
Sut	1.77E+09
Sy	1.64E+09
Preload	4956.23

Bearing Weight (N):	58.86
Weight Gear	135.152

Forces from FBD	Name	Distance
	B1G	95
	B1B2	135
	B1s	0

Force	x	y	z
Fg1	0.00	0.00	10886.00
Fg1a	2916.89	0.00	0.00
Fg1r	0.00	2916.89	0.00
Fws	0.00	0.00	0.00
Frb2	0.00	5293.62	7565.41
Frb1	0.00	-2376.73	3185.44

Points to consider
1-2 shoulder
2 keyway
2-3 shoulder
3-4 large shoulder
5 keyway

Input Shaft Weight						
Section	1	2		3	4	5
		gear				
	bearing				bearing	turbine
Side view						
Feature	*shoulder*	*keyway*	*shoulder*		*shoulder*	*Input Keyway*
Diameter (mm)	35	45	45	55	35	35
Length (mm)	25.4	25.4	0	10	25.4	25.4
Cutouts (mm^2)		63		169.65		40
Cutouts (mm^3)		1280.16		169.65		812.8
I of section (mm^4)	73662	177002	201289	449180	73662	65118
Volume (mm^3)	24438	39117	0	23589	24438	23625
External vertical forces (N) (assumed applied	13374	-22971	0	0	9600	0
Weight + forces (N)	13375.60	-22967.99	0.00	1.82	9601.5	1.82
Moments - Equivalent weight moments at ce	-1	0	0	0	0	0
Moments - Equivalent weight moments at Ec	-234	-218	-218	-218	122	0
Total Weight (N)		12.7				

Input Shaft	
Ft (N)	21772.0
Fa	7924.4
Fr	7924.4
Fw	
Fpre	0

DShaft1	Axial	Bending
Cor. Se:	2.64E+08	3.77E+08
Cload	0.70	1.00
Csize	0.84	0.84
Csurf	0.62	0.62
Ctemp	1.00	1.00
Creliab	0.81	0.81
Se'	8.86E+08	8.86E+08

DShaft2	Axial	Bending
Cor. Se:	2.58E+08	3.68E+08
Cload	0.70	1.00
Csize	0.82	0.82
Csurf	0.62	0.62
Ctemp	1.00	1.00
Creliab	0.81	0.81
Se	8.86E+08	8.86E+08

DShaft3	Axial	Bending
Cor. Se:	2.53E+08	3.61E+08
Cload	0.70	1.00
Csize	0.81	0.81
Csurf	0.62	0.62
Ctemp	1.00	1.00
Creliab	0.81	0.81
Se	8.86E+08	8.86E+08

Figure 9. Input shaft data

Se	3.68E+08
Cor. Mean	3.20E+08
Cor. Amp	9.71E+07
OZ	3.34E+08
ZS	2.00E+08
Cor. Mean @S	3.60E+08
Cor. Amp@S	2.93E+08
Safety Factor	1.60

Point 1: Bearing to Shoulder				
Kt Axial	2.08		Tension	0.00E+00
Kt Moment	1.81		Torsion	1.63E+03
kt Torsional	1.65		Shear	0.00E+00
D/d:	1.29		Moment	-2.34E+02
r/d:	0.06		I (m^4)	7.37E-08
r:	2.00	<-- different	J	1.47E-07
q:	0.92		Area (m^2)	9.62E-04
Kf Axial:	2.08	*<- Kf = Kt*		
Kf Moment:	1.75		*shoulder*	
Kf Torsional:	1.65	*<- Kf = Kt*		

σ Bending nom	-5.56E+07	σ Bending Cor	-9.71E+07
σ Axial nom	0.00E+00	σ Axial Cor.	0.00E+00
τ Torsion Nom	1.94E+08	τ Torsion Cor.	3.20E+08
	σ Mean Vo	3.20E+08	
	σ Alternati	9.71E+07	

Se	3.68E+08
Cor. Mean	2.72E+08
Cor. Amp	5.68E+07
OZ	2.78E+08
ZS	2.50E+08
Cor. Mean @S	3.23E+08
Cor. Amp@S	3.01E+08
Safety Factor	1.90

Point 2: keyway				
Kt Axial	1.00	*assumed*	Tension	0.00E+00
Kt Moment	2.14	*p.607*	Torsion	1.63E+03
kt Torsional	2.62	*p.607*	Shear	
D/d:	1.29		Moment	-2.18E+02
r/d:	0.02		I (m^4)	1.77E-07
r:	1.00		J	3.54E-07
q:	0.92		Area (m^2)	1.59E-03
Kf Axial:	1.00	*<- Kf = Kt*		
Kf Moment:	2.05			
Kf Torsional:	2.62	*<- Kf = Kt*		

σ Bending nom	-2.77E+07	σ Bending Cor	-5.68E+07
σ Axial nom	0.00E+00	σ Axial Cor.	0.00E+00
τ Torsion Nom	1.04E+08	τ Torsion Cor.	2.72E+08
	σ Mean Vo	2.72E+08	
	σ Alternati	5.68E+07	

Se	3.68E+08
Cor. Mean	1.98E+08
Cor. Amp	3.60E+04
OZ	1.98E+08
ZS	3.20E+08
Cor. Mean @S	2.63E+08
Cor. Amp@S	3.14E+08
Safety Factor	2.62

Point 3: Gear to Shoulder				
Kt Axial	2.54		Tension	7.92E+03
Kt Moment	2.23		Torsion	1.63E+03
kt Torsional	1.90		Shear	
D/d:	1.22		Moment	-1.33E-01
r/d:	0.02		I (m^4)	1.77E-07
r:	1.00		J	3.54E-07
q:	0.92		Area (m^2)	1.59E-03
Kf Axial:	2.54	*<- Kf = Kt*		
Kf Moment:	2.13			
Kf Torsional:	1.90	*<- Kf = Kt*		

σ Bending nom	-1.69E+04	σ Bending Cor	-3.60E+04
σ Axial nom	4.98E+06	σ Axial Cor.	1.27E+07
τ Torsion Nom	1.04E+08	τ Torsion Cor.	1.97E+08
	σ Mean Vo	1.98E+08	
	σ Alternati	3.60E+04	

Figure 10. Input shaft safety factor calculations 1

Se	3.68E+08
Cor. Mean	3.75E+08
Cor. Amp	7.51E+07
OZ	3.83E+08
ZS	2.11E+08
Cor. Mean @S	4.18E+08
Cor. Amp@S	2.81E+08
Safety Factor	1.55

Point 4: Large Shoulder to Bearing				
Kt Axial	2.49		Tension	7.92E+03
Kt Moment	2.11		Torsion	1.63E+03
kt Torsional	1.93		Shear	
D/d:	1.29		Moment	1.22E+02
r/d:	0.03		I (m^4)	7.37E-08
r:	1.00		J	1.47E-07
q:	0.92		Area (m^2)	1.59E-03
Kf Axial:	2.49	<- Kf = Kt		
Kf Moment:	2.02			
Kf Torsional:	1.93	<- Kf = Kt		

σ Bending nom	3.72E+07	σ Bending Cor	7.51E+07
σ Axial nom	4.98E+06	σ Axial Cor.	1.24E+07
τ Torsion Nom	1.94E+08	τ Torsion Cor.	3.75E+08
	σ Mean Vc	3.75E+08	
	σ Alternati	7.51E+07	

Se	3.68E+08
Cor. Mean	6.14E+08
Cor. Amp	1.16E+04
OZ	6.14E+08
ZS	2.36E+08
Cor. Mean @S	6.62E+08
Cor. Amp@S	2.31E+08
Safety Factor	1.38

Point 5: Coupling to turbine (keyway)				
Kt Axial	1.00	assumed	Tension	0.00E+00
Kt Moment	1.95	p.607	Torsion	1.63E+03
kt Torsional	2.80	p.607	Shear	
D/d:	1.29		Moment	2.31E-02
r/d:	0.03		I (m^4)	6.51E-08
r:	1.00		J	1.30E-07
q:	0.92		Area (m^2)	9.62E-04
Kf Axial:	1.00	<- Kf = Kt		
Kf Moment:	1.87			
Kf Torsional:	2.80	<- Kf = Kt		

σ Bending nom	6.21E+03	σ Bending Cor	1.16E+04
σ Axial nom	0.00E+00	σ Axial Cor.	0.00E+00
τ Torsion Nom	2.19E+08	τ Torsion Cor.	6.14E+08
	σ Mean Vc	6.14E+08	
	σ Alternati	1.16E+04	

Figure 11. Input shaft safety factor calculations 2

B-3: Intermediate Shaft Calculations

Intermediate Shaft

Torque (Nm)	458.20	Dshaft1	35
Gear Diameter(mm)	300	Shoulder heigh	10
Speed (rpm)	134.71	Dpinion	84.2
Pinion Diameter(mm)	84.18		
Shaft density	7850		
Gear density	7850		
g (m/s^2)	9.81		
Helix Angle	15		
Pressure Angle	15		
Sut	1.77E+09		
Sy	1.64E+09		
Preload	2460.56		

Weight Pinion	7.78
Weight Gear	135.15
Bearing Weight (N)	58.86

Intermediate Shaft Weight							
Section	1	2	3	4	5	6	7
		pinion				gear 2	
	bearing 1						bearing 2
Side view							
Features	shoulder	keyway	shoulder	compression?	shoulder	keyway	shoulder
Diameter (mm) (DShaft1,DShaft2,DShaft3)	35	45	45	55	45	45	35
Length (mm)	25.4	25.4	0	55	0	25.4	25.4
Cutouts area (mm^2)		63				63	
Cutouts volume (mm^3)		1280.16				1280.16	
I of section (mm^4)	73662	177002	201289	449180	201289	177002	73662
Volume (mm^3)	24438	39117	0	130671	0	39117	24438
External Forces (N) (vertical)(assumed applied at centre of section)	8603.93	11270.02	0.00	0.00	0.00	3162.44	1247.16
Weight (N)	1.88	3.01	0.00	10.06	0.00	3.01	1.88
Weight + Forces (N)	8605.82	11273.03	0.00	10.06	0.00	3165.46	1249.04
Moments - Equivalent weight moments at centre of section (N*m)	-786	-168	779	1447	2503	3012	3680
Moments - Equivalent weight moments at Edge POI(N*m) (z-axis)	-151	-471	-471	-1018	-1564	-1564	-2110
Total Weight (N)		19.85					

Lengths from FBD	
Name	Distance
B1G1	25.4
B1G2	105.8
B1B2	131.2

Forces from FBD			
Force	x	y	z
Reaction B1	0.00	2510.7	8229.5
Reaction B2	0.00	1224.7	-235.4
Gear 1	2916.89	2916.9	10886.0
Gear 2	818.50	818.5	3054.7
Weight G1	0.00	0.0	7.8
Weight G2	0.00	0.0	135.2
Weight Shaft	0.00	0	19.85

Points to consider
1-2 shoulder
2 keyway
2-3 shoulder
3 compression?
3-4 shoulder
4 keyway
4-5 shoulder

DShaft1	Axial	Bending
Cor. Se:	2.64E+08	3.77E+08
Cload	0.70	1.00
Csize	0.84	0.84
Csurf	0.62	0.62
Ctemp	1.00	1.00
Creliab	0.81	0.81
Se'	8.86E+08	8.86E+08

DShaft2	Axial	Bending
Cor. Se:	2.58E+08	3.68E+08
Cload	0.70	1.00
Csize	0.82	0.82
Csurf	0.62	0.62
Ctemp	1.00	1.00
Creliab	0.81	0.81
Se	8.86E+08	8.86E+08

DShaft3	Axial	Bending
Cor. Se:	2.53E+08	3.61E+08
Cload	0.70	1.00
Csize	0.81	0.81
Csurf	0.62	0.62
Ctemp	1.00	1.00
Creliab	0.81	0.81
Se	8.86E+08	8.86E+08

Figure 12. Intermediate shaft data

39	Se	2.64E+08	Point 1: Left Bearing to Gear Shoulder					Loading	
40	Cor. Mean	0.00E+00	Kt Axial	2.50		Tension	0.00E+00	Mz1	3.19E+01
41	Cor. Amp	5.57E+07	Kt Moment	2.11		Torsion	0.00E+00	My1	1.05E+02
42	OZ	5.57E+07	kt Torsional	1.93		Shear		σ Bending Max 1	2.60E+07
43	ZS	2.06E+08	D/d:	1.29		Moment	1.09E+02	σ Bending Cor 1	5.57E+07
44	Cor. Mean @S	3.04E+07	r/d:	0.03		I (m^4)	7.37E-08	τ Torsion Nom 1	0.00E+00
45	Cor. Amp@S	2.60E+08	r:	1.00		J	1.47E-07	τ Torsion Cor. 1	0.00E+00
46	Safety Factor	4.70	q:	0.92		Area (m^2)	9.62E-04	σ Axial nom	0.00E+00
47			Kf Axial:	2.50	<- Kf = Kt			σ Axial Cor.	0.00E+00
48			Kf Moment:	2.02				σ Mean Von Mises	0.00E+00
49			Kf Torsional:	1.93	<- Kf = Kt			σ Amplitude Von Mises	5.57E+07
50									
51	Se	3.68E+08	Point 2 (left keyway)					Loading	
52	Cor. Mean	1.32E+08	Kt Axial	1.00	assumed	Tension	0.00E+00	Mz2	-5.90E+01
53	Cor. Amp	4.33E+07	Kt Moment	2.14	p.607	Torsion	4.58E+02	My2	-2.09E+02
54	OZ	1.39E+08	kt Torsional	2.62	p.607	Shear		σ Bending Max 2	2.76E+07
55	ZS	2.91E+08	D/d:	1.33		Moment	2.17E+02	σ Bending Cor 2	4.33E+07
56	Cor. Mean @S	1.91E+08	r/d:	0.02		I (m^4)	1.77E-07	τ Torsion Nom 2	2.91E+07
57	Cor. Amp@S	3.29E+08	r:	1.00		J	3.54E-07	τ Torsion Cor. 2	7.63E+07
58	Safety Factor	3.09	q:	0.50		Area (m^2)	1.59E-03	σ Axial nom	0.00E+00
59			Kf Axial:	1.00	<- Kf = Kt			σ Axial Cor.	0.00E+00
60			Kf Moment:	1.57				σ Mean Von Mises	1.32E+08
61			Kf Torsional:	2.62	<- Kf = Kt			σ Amplitude Von Mises	4.33E+07
62									
63	Se	3.68E+08	Point 3 (inner left shoulder)					Loading	
64	Cor. Mean	9.09E+07	Kt Axial	2.66		Tension	0.00E+00	Mz3	1.54E+02
65	Cor. Amp	3.80E+07	Kt Moment	2.23		Torsion	4.58E+02	My3	4.27E+01
66	OZ	9.85E+07	kt Torsional	2.05		Shear		σ Bending Max 3	1.78E+07
67	ZS	3.05E+08	D/d:	1.44		Moment	1.60E+02	σ Bending Cor 3	3.80E+07
68	Cor. Mean @S	1.53E+08	r/d:	0.02		I (m^4)	2.01E-07	τ Torsion Nom 3	2.56E+07
69	Cor. Amp@S	3.37E+08	r:	1.00		J	4.03E-07	τ Torsion Cor. 3	5.25E+07
70	Safety Factor	4.09	q:	0.92		Area (m^2)	1.59E-03	σ Axial nom	0.00E+00
71			Kf Axial:	2.66	<- Kf = Kt			σ Axial Cor.	0.00E+00
72			Kf Moment:	2.13				σ Mean Von Mises	9.09E+07
73			Kf Torsional:	2.05	<- Kf = Kt			σ Amplitude Von Mises	3.80E+07
74									
75	Se	3.61E+08	Point 4 (compression? unlikely)					Loading	
76	Cor. Mean	2.43E+07	Kt Axial	1.00		Tension	0.00E+00	Mz4	-7.53E+01
77	Cor. Amp	7.76E+06	Kt Moment	1.00		Torsion	4.58E+02	My4	-1.02E+02
78	OZ	2.55E+07	kt Torsional	1.00		Shear		σ Bending Max 4	7.76E+06
79	ZS	3.41E+08	D/d:	1.00		Moment	1.27E+02	σ Bending Cor 4	7.76E+06
80	Cor. Mean @S	9.25E+07	r/d:			I (m^4)	4.49E-07	τ Torsion Nom 4	1.40E+07
81	Cor. Amp@S	3.42E+08	r:	0.00		J	8.98E-07	τ Torsion Cor. 4	1.40E+07
82	Safety Factor	14.38	q:	1.00		Area (m^2)	2.38E-03	σ Axial nom	1.04E+06
83			Kf Axial:	1.00	<- Kf = Kt			σ Axial Cor.	1.04E+06
84			Kf Moment:	1.00				σ Mean Von Mises	2.43E+07
85			Kf Torsional:	1.00	<- Kf = Kt			σ Amplitude Von Mises	7.76E+06

Figure 13. Intermediate shaft safety factors 1

87	Se	3.68E+08	Point 5 (inner right shoulder)					Loading	
88	Cor. Mean	9.09E+07	Kt Axial	2.66		Tension	0.00E+00	Mz5	8.65E+01
89	Cor. Amp	2.16E+07	Kt Moment	2.23		Torsion	4.58E+02	My5	2.81E+01
90	OZ	9.35E+07	kt Torsional	2.05		Shear		σ Bending Max 5	1.02E+07
91	ZS	3.21E+08	D/d:	1.44		Moment	9.10E+01	σ Bending Cor 5	2.16E+07
92	Cor. Mean @S	1.56E+08	r/d:	0.02		I (m^4)	2.01E-07	τ Torsion Nom 5	2.56E+07
93	Cor. Amp@S	3.36E+08	r:	1.00		J	4.03E-07	τ Torsion Cor. 5	5.25E+07
94	Safety Factor	4.43	q:	0.92		Area (m^2)	1.59E-03	σ Axial nom	0.00E+00
95			Kf Axial:	2.66	<- Kf = Kt			σ Axial Cor.	0.00E+00
96			Kf Moment:	2.13				σ Mean Von Mises	9.09E+07
97			Kf Torsional:	2.05	<- Kf = Kt			σ Amplitude Von Mises	2.16E+07
98									
99	Se	3.68E+08	Point 6 (keyway)					Loading	
100	Cor. Mean	1.32E+08	Kt Axial	1.00	assumed	Tension	0.00E+00	Mz6	9.17E+01
101	Cor. Amp	1.83E+07	Kt Moment	2.14	p.607	Torsion	4.58E+02	My6	-5.98E+00
102	OZ	1.33E+08	kt Torsional	2.62	p.607	Shear		σ Bending Max 6	1.17E+07
103	ZS	3.16E+08	D/d:	1.29		Moment	9.19E+01	σ Bending Cor 6	1.83E+07
104	Cor. Mean @S	1.96E+08	r/d:	0.02		I (m^4)	1.77E-07	τ Torsion Nom 6	2.91E+07
105	Cor. Amp@S	3.27E+08	r:	1.00		J	3.54E-07	τ Torsion Cor. 6	7.63E+07
106	Safety Factor	3.37	q:	0.50		Area (m^2)	1.59E-03	σ Axial nom	0.00E+00
107			Kf Axial:	1.00	<- Kf = Kt			σ Axial Cor.	0.00E+00
108			Kf Moment:	1.57				σ Mean Von Mises	1.32E+08
109			Kf Torsional:	2.62	<- Kf = Kt			σ Amplitude Von Mises	1.83E+07
110									
111	Se	2.64E+08	Point 7 (right gear to bearing shoulder)					Loading	
112	Cor. Mean	0.00E+00	Kt Axial	2.50		Tension	0.00E+00	Mz7	-1.56E+01
113	Cor. Amp	7.60E+06	Kt Moment	2.11		Torsion	0.00E+00	My7	-2.99E+00
114	OZ	7.60E+06	kt Torsional	1.93		Shear		σ Bending Max 7	3.76E+06
115	ZS	2.54E+08	D/d:	1.29		Moment	1.58E+01	σ Bending Cor 7	7.60E+06
116	Cor. Mean @S	3.74E+07	r/d:	0.03		I (m^4)	7.37E-08	τ Torsion Nom 7	0.00E+00
117	Cor. Amp@S	2.59E+08	r:	1.00		J	1.47E-07	τ Torsion Cor. 7	0.00E+00
118	Safety Factor	34.41	q:	0.92		Area (m^2)	9.62E-04	σ Axial nom	0.00E+00
119			Kf Axial:	2.50	<- Kf = Kt			σ Axial Cor.	0.00E+00
120			Kf Moment:	2.02				σ Mean Von Mises	0.00E+00
121			Kf Torsional:	1.93	<- Kf = Kt			σ Amplitude Von Mises	7.60E+06
122									

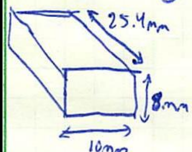
Figure 14. Intermediate shaft safety factors 2

B-4: Key Failure Calculations

Key Failure Sample Calculation.

Shear Failure (key sheared) Bearing Failure (key deformed)

Input Key: $F = T/(D/2) = \dfrac{(1632.9\ N\cdot m)}{(0.035m/2)} = 93.3\ kN$



$A_{shear} = (0.01m)(0.0254m) = 2.54\times10^{-4}\ m^2$

$A_{bearing} = \left(\dfrac{0.008m}{2}\right)(0.0254m) = 1.016\times10^{-4}\ m^2$

$\text{Shear} = \tau_{xy} = F/A_{shear} = \dfrac{93.3kN}{2.54\times10^{-4}m^2} = 367.3\ MPa$

$\text{Bearing stress} = \sigma_x = F/A_{bearing} = \dfrac{93.3kN}{1.016\times10^{-4}m^2} = 918.3\ MPa$

$\text{Shear failure Safety Factor} = \dfrac{S_{ys}}{\tau_{xy}} = \dfrac{\frac{1}{2}S_y}{\tau_{xy}} = \dfrac{\frac{1}{2}(1.64\times10^9\ Pa)}{367.3\ MPa} = 2.23$

$\text{Bearing failure Safety Factor} = \dfrac{S_y}{\sigma_x} = \dfrac{(1.64\times10^9\ Pa)}{918.3\ MPa} = 1.786$

Figure 15. Sample key failure calculations

B-5: Gears and Pinions Calculations

Gear and Pinion Specifications:	Match Table?
modulus $= \frac{2 \cdot \text{pitch radius}}{N} = \frac{2 \cdot 150}{125} = 2.400\ mm$	✓
addendum $= 1 \cdot$ modulus $= 2.400\ mm$	✓
dedendum $= 1.25 \cdot$ modulus $= 3.000\ mm$	✓
Lenght of Action $= \sqrt{(r_p+a_p)^2-(r_p \cos\phi)^2} + \sqrt{(r_g+a_g)^2-(r_g \cos\phi)^2} - C\cos\phi =$	
$= \sqrt{(42.05+2.4)^2-(42.05 \cos 15)^2} + \sqrt{(150+2.4)^2-(150 \cos 15)^2} - (150+42.05)\cos 15 =$	
$= 41.569$	✓
Contact Ratio $= m_p = \frac{Z}{m \cdot \pi \cdot \cos\phi} = \frac{41.569}{2.4 \cdot \pi \cdot \cos(15)} = 5.708$	✓
Helix angle is 15° and Pressure angle is 15°	✓
Gear Quality is 9 for all gears.	✓

Figure 16 Gear and pinion specifications sample calculations

Gear and Pinion Sample analysis:

Match Table?

Bending:

$$\sigma_b = \frac{W_t \cdot K_a \cdot K_m \cdot K_s \cdot K_B \cdot K_i}{F \cdot m \cdot J \cdot K_v}$$

Finding the variables: From outlined conditions and assumptions: $W_t = 10886$ N

$K_m = 1.6$ $K_s = 1.0$ $K_i = 1.0$ $m = 2.4$ mm ✓

$K_a = 1.0$ $K_b = 1.$ $J = 0.5$ $F = 25.4$ mm ✓

$$B = \frac{(12-Q_v)^{2/3}}{4} = \frac{(12-12)^{2/3}}{4} = 0.$$ ✓

$$A = 50+56(1-B) = 50+56(1-0) = 106$$ ✓

$$K_v = \left(\frac{A}{A+\sqrt{200\cdot V_t}}\right)^{B} = \left(\frac{106}{106+\sqrt{200\cdot 34.34}}\right)^{0} = 1$$ ✓

$$V_t = \left[\frac{A+(Q_v-3)}{200}\right]^2 = \left[\frac{106+(12-3)}{200}\right]^2 = 66.125$$ ✓

$$\sigma_b = \frac{(10886)(1.0)(1.0)(1.0)(1.0)(1.0)}{(25.4)(2.4)(0.5)(1)(10^{-6})} = 571.4\text{ MPa}$$ ✓

$S_{fb'} = 520$ MPa

$$S_{fb} = \frac{K_L \cdot S_{fb'}}{K_T K_R}$$ $K_T = 1.0$, $K_R = 1.0$

from Figure 12-24 [1]

$$K_L = 6.1514 N^{-0.1192} = 6.1514(6\times10^{4})^{-0.1192} = 1.657$$ ✓

$$S_{fb} = \frac{1.657 \cdot 520}{1 \cdot 1} = 861.8\text{ MPa}$$ ✓

$$N_b = \frac{S_{fb}}{\sigma_b} = \frac{861.8}{571.4} = 1.508$$ ✓

Figure 17 Sample calculations on the input gear for bending failure.

Surface: | Match Table

$$\sigma_c = C_p \sqrt{\frac{W_t \cdot C_a \cdot C_m \cdot C_s \cdot C_f}{F I d \cdot C_v}}$$

$C_v = K_v = 1.0$, $C_m = K_m = 1.6$, $C_a = K_a = 1.0$, $C_s = K_s = 1.0$ ✓

$C_f = 1.0$ ✓

$W_t = 10886\,N$ $\quad d = 84.2\,mm$ ✓

$$C_p = \sqrt{\frac{1}{\pi\left[\left(\frac{1-\nu_p^2}{E_p}\right) + \left(\frac{1-\nu_g^2}{E_g}\right)\right]}} = \sqrt{\frac{1}{\pi\left[\left(\frac{1-0.28^2}{2E5}\right)+\left(\frac{1-0.28^2}{2E5}\right)\right]}} =$$

$C_p = 185.8$ ✓

$\phi_n = \tan^{-1}(\tan\phi \cdot \cos\psi) = \tan^{-1}(\tan(15)\cos(15)) = 14.5°$ ✓

$$\psi_b = \cos^{-1}\left(\cos\psi \cdot \left(\frac{\cos\phi_n}{\cos\phi}\right)\right) = \cos^{-1}\left(\cos(15) \cdot \left(\frac{\cos(14.5)}{\cos(15)}\right)\right) = 14.5°$$ ✓

$$m_F = \frac{F \cdot \tan\psi}{m\pi} = \frac{25.4 \tan(15)}{2.4 \cdot \pi} = 0.9027$$ ✓

$n_a = 0.9027$, $n_r = 0.7078$, $n_a > 1 - n_r$ ✓

$$L_{min} = \frac{m_p F - (1-n_a)(1-n_r)\, m\pi}{\tan(\psi)\cos(\psi_b)} =$$

$$= \frac{5.708 \cdot 25.4 - (1-0.9027)(1-0.7078)(2.4)\pi}{\tan(15)\cos(15)} =$$

$L_{min} = 144.15\,mm$ ✓

$$m_N = \frac{F}{L_{min}} = \frac{25.4}{144.15} = 0.1762$$ ✓

$$\rho_p = \sqrt{[0.5((r_p+a_p)+(C-r_g-a_g))]^2 - (r_p\cos\phi)^2} =$$

$$\rho_p = \sqrt{\{0.5[(42.05 \cdot 2)]^2 - (42.05\cos(15))^2} = 10.89\,mm$$ ✓

Figure 18 Gear 1 sample calculations for surface failure, part 1

Surface :(Cont.) | Match Table

$\rho_g = (r_p + r_g)\sin(\phi) - \rho_r = (150 + 42.09)\sin(15) - 10.89 = 38.82\ mm$ ✓

$$I = \frac{\cos\phi}{\left(\frac{1}{\rho_p} + \frac{1}{\rho_g}\right) d_p m_N} = \frac{\cos 15}{\left(\frac{1}{10.89} + \frac{1}{38.82}\right) 84.2 \cdot 0.176} = 0.554$$ ✓

$$\sigma_c = 185.8\sqrt{\frac{10886(1.6)(1)(1)(1)}{(25.4)\cdot(0.554)(84.2)(1)}} = 712.66\ MPa$$ ✓

$S_{fc}' = 1300\ MPa$ ✓

$S_{fc} = \frac{C_L C_H S_{fc}'}{C_T C_R}$ $\quad C_T = K_T = 1,\ C_R = K_R = 1 \quad C_H = 1$ ✓

$C_L = 2.466(N)^{-0.056} = 2.466(6 \cdot 10^{4})^{-0.056} = 1.331$ ✓

$S_{fc} = 1.331 \cdot 1300 = 1731\ MPa$ ✓

$$N_c = \left(\frac{S_{fc}}{\sigma_c}\right)^2 = \left(\frac{1731}{712.66}\right)^2 = 5.901$$ ✓

Figure 19. Part 2 of surface failure sample calculations on Gear 1

Input Gear

General Data	Bending	Surface
Face Width	25.4	
Modulus	2.4	
Geometry Factor(J, I)	0.5	5.54E-01
Dynamic Factor(Kv, Cv)	1	1
Load Distribution(Km, Cm)	1.6	1.6
Application Factor(Ka, Ca)	1	1
Size Factor(Ks, Cs)	1	1
Rim Thickness Factor(Kb)	1	NA
Idler factor(Ki)	1	NA
Elastic Coefficient (Cp)	NA	185.8463
Surface Finish Factor(Cf)	NA	1
Wt	10886	
Stress	571.4435696	712.66

Safety factors	Bending	Surface
Fatigue Strength (uncorected)	520	1300
Life Factor(Kl,Cl)	1.657365672	1.331731
Temperature Factor(Kt,Ct)	1	1
Reliability Factor(Kr,Cr)	1	1
Hardness Ratio Factor(Ch)	NA	1
Corrected Fatigue Strength	861.8301493	1731.251
Safety Factor	1.508163177	5.901401

ro(g):	38.82285677
ro(p):	10.89395631
A:	106
B:	0
Vt:	66.125
v (Poisson)	0.28
Modulus of Elasticit	2.00E+05
A:	106
B:	0
Vt:	66.125

Variables for I:	
Axial contact ratio(mf)	0.902661781
na	0.902661781
nr	0.707784727
Lmin	144.1509813
theta,n	14.5108187
phi,b	14.5108187
mn	0.176204142

Service Intervals(hr):	
Gear 1:	2.65E+01
Pinion 1:	1.24E+06
Gear 2:	1.24E+06
Pinion 2:	3.47E+05

Figure 20. Input gear fatigue analysis and safety factor data

First Pinion

General Data	Bending	Surface
Face Width	25.4	
Modulus	2.4	
Geometry Factor(J,I)	0.47	0.55396
Dynamic Factor(Kv, Cv)	1	1
Load Distribution(Km, Cm)	1.6	1.6
Application Factor(Ka, Ca)	1	1
Size Factor(Ks, Cs)	1	1
Rim Thickness Factor(Kb)	1	NA
Idler factor(Ki)	1	NA
Elastic Coefficient (Cp)	NA	185.8463
Surface Finish Factor(Cf)	NA	1
Wt	2916.894909	
Stress	162.8913223	195.4149

Safety factors	Bending	Surface
Fatigue Strength (uncorected)	520	1300
Life Factor(Kl,Cl)	0.899902855	0.679193
Temperature Factor(Kt,Ct)	1	1
Reliability Factor(Kr,Cr)	1	1
Hardness Ratio Factor(Ch)	NA	1
Corrected Fatigue Strength	467.9494845	882.9506
Safety Factor	2.872771108	20.41538

A:	106
B:	0
Vt:	66.125
v (Poisson)	0.28
Modulus of Elasticit	2.00E+05
A:	106
B:	0
Vt:	66.125

Second Gear

General Data	Bending	Surface
Face Width	25.4	
Modulus	2.4	
Geometry Factor(J,I)	0.5	0.155445
Dynamic Factor(Kv, Cv)	1	1
Load Distribution(Km, Cm)	1.6	1.6
Application Factor(Ka, Ca)	1	1
Size Factor(Ks)	1	1
Rim Thickness Factor(Kb)	1	NA
Idler factor(Ki)	1	NA
Elastic Coefficient (Cp)	NA	185.8463
Surface Finish Factor(Cf)	NA	1
Wt	3054.685262	
Stress	160.3509324	377.5126

ro(g):	38.82285677
ro(p):	10.89395631
A:	106
B:	0
Vt:	66.125
v (Poisson)	0.28
Modulus of Elasticit	2.00E+05
A:	106
B:	0
Vt:	66.125

Safety factors	Bending	Surface
Fatigue Strength (uncorected)	520	1300
Life Factor(Kl,Cl)	0.899902855	0.679193
Temperature Factor(Kt,Ct)	1	1
Reliability Factor(Kr,Cr)	1	1
Hardness Ratio Factor(Ch)	NA	1
Corrected Fatigue Strength	467.9494845	882.9506
Safety Factor	2.918283527	5.470285

Figure 22. Intermediate gear fatigue analysis and safety factor data

B-6: Bearings Calculations

Bearing Sample Calculations

Bearing #3

$F_y = 2510\ N \quad F_z = 8229\ N$

$F_r = \sqrt{F_y^2 + F_z^2} = \sqrt{2510^2 + 8229^2} = 8603\ N$ Matches Spreadsheet ✓

Internal Axial Force $= \frac{F_r}{Y} = \frac{8603\ N}{1.6} = 5377\ N$ ✓

$F_a = 2916\ N$

Required Preload $= |IAF - F_a| = 2461$ ✓

$e = \frac{F_a}{VF_R} = \frac{2916\ N}{(1)\,8603\ N} = 0.339$ ✓

$0.339 < 0.37 \therefore P = F_R = 8603\ N$ ✓

$L_{10} = K_r \left(\frac{C}{P}\right)^{10/3} = 0.21 \left(\frac{81{,}200\ N}{8603\ N}\right)^{10/3} = 373 \times 10^6\ \text{cycles}$

$\text{Service Interval} = \frac{L_{10}(10^6)}{60\,(\text{Shaft RPM})} = \frac{373 \times 10^6}{60\,(134.7\ \text{RPM})} = 46{,}152\ \text{Hr}$

Figure 23: Bearing sample calculations

B-7: Fastener Calculations

Fastener Sample Calculations: | Match Table:

Yielding failure:

$F_i = 0.9\, S_p\, A_t = 0.9 \cdot 3.80\times10^{8} \cdot 14.18\times10^{-3} = 4849.56$ ✓

$S_p = 3.80\times10^{8}\ Pa$ ✓

$A_t = 14.18\ mm = 14.18\times10^{-3}\ m$ ✓

$d = 5\ mm \quad l_{bolt} = l = 40\ mm,\ E_b = 2E5\ MPa$ ✓

$l_{thd} = 2d + 6 = 2\cdot5 + 6 = 16\ mm = l_t$ ✓

$l_s = l - l_t = 40 - 16 = 24\ mm$ ✓

$$k_b = \left(1+\frac{d}{l}\right)^{-1} \frac{A_t A_b}{A_b l_t + A_t l_s} E_b = \left(1+\frac{5}{40}\right)^{-1} \frac{14.18\cdot 5^2\pi/4}{\frac{5^2\pi\cdot 16}{4} + 14.18\cdot 24}$$

$$= 7.56\times10^{10}$$ ✓

$$k_m = k_b\cdot\frac{(1-C)}{C} = 7.56\times10^{10}\frac{(1-0.2145)}{0.2145} = 2.77E11$$ ✓

$$j = \frac{5}{40} = 0.125 \approx \underline{1} \qquad r = \frac{69\times10^{9}}{2\times10^{11}} = 0.345$$ ✓

Using table 15-8 [1]: $j = 1$

$p_0 = 0.4389,\ p_1 = -0.9197,\ p_2 = 0.8901,\ p_3 = -0.3187$

$$C = C_r = p_3 r^3 + p_2 r^2 + p_1 r + p_0 =$$

$$= -0.3187\cdot(0.345)^3 + 0.8901(0.345)^2 + (-0.9197)(0.345) + 0.4389 =$$

$C = 0.2145$ ✓

$$P = \frac{F_{applied}}{\#bolts} = \frac{4956}{4} = 1239\ N$$ ✓

$P_b = P\cdot C = 1239\cdot 0.2145 = 265.7\ N$ ✓

$P_m = (1-C)P = (1-0.2145)(1239) = 973.3\ N$ ✓

$F_b = P_b + F_i = 265.7 + 4849.56 = 5115\ N$ ✓

$F_m = F_i - P_m = 4849.56 - 973.3 = 3876.2\ N$ ✓

Figure 24. Endcap fastener sample calculations for yielding

Yielding Failure Cont: Match Table

$$\sigma_y = \frac{F_b}{A_t} = \frac{5115}{14.18\times10^{-6}} = 3.607\times10^{8}\,Pa$$ ✓

$$SF_y = \frac{S_y}{\sigma_y} = \frac{4.20\times10^{8}}{3.607\times10^{8}} = \underline{\underline{1.164}}$$ ✓

Separation:

$$P_o = \frac{F_i}{(1-C)} = \frac{4849.56}{(1-0.2145)} = 6173.5\,N$$ ✓

$$SF_{separation} = \frac{P_o}{P_b} = \frac{6173.5}{1239} = \underline{4.982}$$ ✓

Figure 25. Endcap fastener sample calculations

Input/Output shaft end cap(4 bolts)							
				Forces:		Strengths:	
Bolt diameter	5	MoE	2E+11	Preload(Fi)	4849.56	Class	5.8
Threaded length (Lthd)	16	Stiffness of the bolt (kb')	75628885751	X-dir force/ # bolts(P)	1239.05656	Proof (Sp)	3.80E+08
Total length of the bolt (Lbolt)	40	Material stiffness(km)	2.77018E+11	Pb	265.728906	Yield (Sy)	4.20E+08
Shank length (Ls)	24	Joint's Stiffness constant (C)	0.214460675	Pm	973.327651	Tensile(Sut)	5.20E+08
Length of thread (Lt)	16	j	0.125	Fb	5115.28891		
Total length of material (L)	40	r	0.345	Fm	3876.23235	Safety Factors:	
Tensile stress Area (At)	14.18	po	0.4389	Po	6173.54198	Tensile stress bolt (sigma,b)	360739697
		p1	-0.9197			Safety factor(SF)	1.1642744
		p2	0.8901			Safety factor separation(Nsep)	4.9824537
		p3	-0.3187				

Figure 26. Input and output endcap fastener data

Intermediate shaft end cap(5 bolts,input side)							
				Forces:		Strengths:	
Bolt diameter	6	MoE	2E+11	Preload(Fi)	6881.04	Class	5.8
Threaded length (Lthd)	12.5	Stiffness of the bolt (kb')	50297276331	X-dir force/ # bolts(P)	492.112796	Proof (Sp)	3.80E+08
Total length of the bolt (Lbolt)	40	Material stiffness(km)	1.84232E+11	Pb	105.538842	Yield (Sy)	4.20E+08
Shank length (Ls)	27.5	Joint's Stiffness constant (C)	0.214460675	Pm	386.573953	Tensile(Sut)	5.20E+08
Length of thread (Lt)	12.5	j	0.15	Fb	6986.57884		
Total length of material (L)	40	r	0.345	Fm	6494.46605	Safety Factors:	
Tensile stress Area (At)	20.12	po	0.4389	Po	8759.63785	Tensile stress bolt (sigma,b)	347245469
		p1	-0.9197			Safety factor(SF)	1.209519
		p2	0.8901			Safety factor separation(Nsep)	17.800061
		p3	-0.3187				

Figure 27. Intermediate shaft input endcap data

Intermediate shaft end cap(4 bolts, output side)							
				Forces:		Strengths:	
Bolt diameter	5	MoE	2E+11	Preload(Fi)	4849.56	Class	5.8
Threaded length (Lthd)	16	Stiffness of the bolt (kb')	30994908243	X-dir force/ # bolts(P)	9.75654494	Proof (Sp)	3.80E+08
Total length of the bolt (Lbolt)	40	Material stiffness(km)	1.1353E+11	Pb	2.09239522	Yield (Sy)	4.20E+08
Shank length (Ls)	24	Joint's Stiffness constant (C)	0.214460675	Pm	7.66414973	Tensile(Sut)	5.20E+08
Length of thread (Lt)	16	j	0.125	Fb	4851.6524		
Total length of material (L)	40	r	0.345	Fm	4841.89585	Safety Factors:	
Tensile stress Area (At)	14.18	po	0.4389	Po	6173.54198	Tensile stress bolt (sigma,b)	342147560
		p1	-0.9197			Safety factor(SF)	1.2275405
		p2	0.8901			Safety factor separation(Nsep)	632.75904
		p3	-0.3187				

Figure 28. Intermediate shaft output endcap data

Case Cap(14 bolts)							
				Forces:		Strengths:	
Bolt diameter	4	MoE	2E+11	Preload(Fi)	3002.76	Class	5.8
Threaded length (Lthd)	14	Stiffness of the bolt (kb')	24311036360	X-dir force/ # bolts(P)	0	Proof (Sp)	3.80E+08
Total length of the bolt (Lbolt)	20	Material stiffness(km)	51319816509	Pb	0	Yield (Sy)	4.20E+08
Shank length (Ls)	6	Joint's Stiffness constant (C)	0.321443372	Pm	0	Tensile(Sut)	5.20E+08
Length of thread (Lt)	14	j	0.2	Fb	3002.76		
Total length of material (L)	20	r	0.345	Fm	3002.76	Safety Factors:	
Tensile stress Area (At)	8.78	po	0.6118	Po	4425.21652	Tensile stress bolt (sigma,b)	342000000
		p1	-1.1715			Safety factor(SF)	1.2280702
		p2	1.0875			Safety factor separation(Nsep)	N/A
		p3	-0.3806				

Figure 29. Gearbox case cap data

Mounting Reactions

Simplifying Assumptions

→ Left Right Reactions are split equally between Bolts leaving R_1 and R_2 as the reactions transferred to bed plate

→ Center of Mass of GB is Approx in the Middle

$T_i - T_o = (1632\,Nm - 117\,Nm) = 1515\,Nm$

$\overrightarrow{R_1R_2} = 521\,mm$ $\qquad \overrightarrow{R_2T} = 521 - 162 = 359\,mm$

$\overrightarrow{R_1T} = 162\,mm$ $\qquad W_{GB} = 68\,kg(9.81\,ms^{-2})$

$\overrightarrow{R_1T_2} = 176\,mm$ $\qquad = 666.4\,N$

① $\sum M_{(T's)} = 0 = 1515\,Nm + W_{GB}(\frac{521}{2} - 162) - R_2(R_2T) - R_1(R_1T)$

② $\sum F_{vert} = 0 = R_2 + W_{GB} - R_1 = 0$

$\rightarrow R_1 = R_2 - W_{GB}$

② → ① ⇒ $1515\,Nm + W_{GB}(98.5\,mm) - R_2(359\,mm) - (R_2 - W_{GB})(162\,mm)$

$R_2 \Rightarrow 1515\,Nm + 666.1\,N(98.5\,mm) - R_2(359\,mm) - R_2(162\,mm) + 666.1\,N(162\,mm)$

$$R_2 = \frac{1515\,Nm + 666.1\,N(0.0985 + 0.162)\,m}{(0.359 + 0.162)\,m}$$

$= 3240\,N$

$\Rightarrow R_1 = 3240\,N - 666.1\,N = 2575\,N$

Figure 30. Mounting reaction forces

Static Bolt Failure

Speced Bolt: M8 Class 8.8 (Coarse)

Clamped Length (L_c): 12mm

$S_{proof} = 380\ MPa$ $E_{Bolt} = 206.8\ GPa$ $E_{case} = 165\ MPa$

Tensile Stress Area $(A_t) = 36.61\ mm^2$

Preload Force (F_i): $0.9\,(S_p) A_t = 0.9(600\,MPa)(36.61\,mm^2)$

$= 19.8\ KN$

Assumption: Threaded Whole Length - Short bolt

$$K_b: \frac{A_t E_{Bolt}}{l} = \frac{(36.61\,mm^2)(206.8\,GPa)}{12\,mm} = 631\ MN/m$$

$$K_m = \frac{A_m E_{case}}{l} = \frac{\pi/4\,(17\,mm^2 - 9\,mm^2)\,165\,MPa}{12\,mm} = 4.72\ MN/m$$

$$C = \frac{k_b}{k_m + k_b} = 0.9926$$

$$P_b = CP = 0.9926(P)$$

Case 1: Input/Output Side

$P = R_1 / \#\text{ of Bolts}$

$= 2575\,N / 2\text{ Bolts}$

$= 1287.5\,N$

$P_b = 1278\,N$

$F_b = F_i + P_b = 21.1\ KN$

$\sigma_b = F_b / A_t = 575.7\ MPa$

$$N_y = \frac{S_y}{\sigma_b} = \frac{660\,MPa}{576\,MPa} = 1.15$$

$$N_{sep} = \frac{F_i}{(1-C)P} = 1.31$$

Case 2: Intermediate Shaft Side

$P = R_2 / \#\text{ of Bolts}$

$= 3240\,N / 2\text{ Bolts}$

$= 1620\,N$

$P_b = 1608\,N$

$F_b = 21.4\ KN$

$\sigma_b = 584.8\ MPa$

$$N_y = \frac{660\,MPa}{585\,MPa} = 1.13$$

$N_{sep} = 1.042$

Figure 31. Bedplate bolt analysis

Appendix C: Drawings and CAD

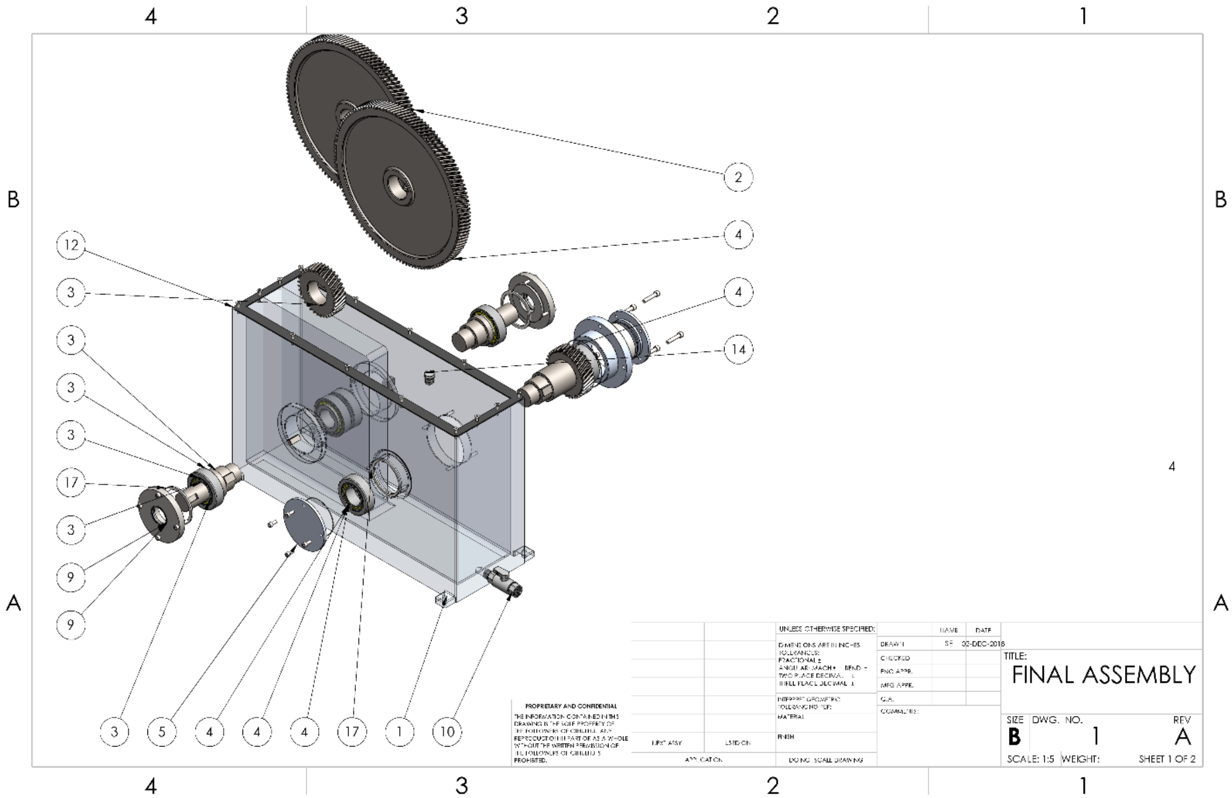


Figure 32. Assembly drawing exploded view

ITEM NO.	PART NUMBER	DESCRIPTION	QTY.
1	Gearbox		1
2	Input shaft Assembly		1
	Large Gear DRAFT		1
	32207		2
	32207_ir		1
	32207_cage		1
	32207_roller		17
	32207_or		1
	Input output Shaft		1
	Key 3-16in		1
	Small key 10x8		1
3	Output Shaft Assembly		1
	Input output Shaft		1
	Key 3-16in		1
	Small Gear DRAFT		1
	32207		2
	32207_ir		1
	32207_cage		1
	32207_roller		17
	32207_or		1
	Small key 10x8		1
4	INTERMEDIATE SHAFT ASSEMBLY]		1
	Large Gear DRAFT		1
	Secondary Shaft		1
	Small Gear DRAFT		1
	Key 3-16in		2
	32207		2
	32207_ir		1
	32207_cage		1
	32207_roller		17
	32207_or		1
5	Secondary cap		1

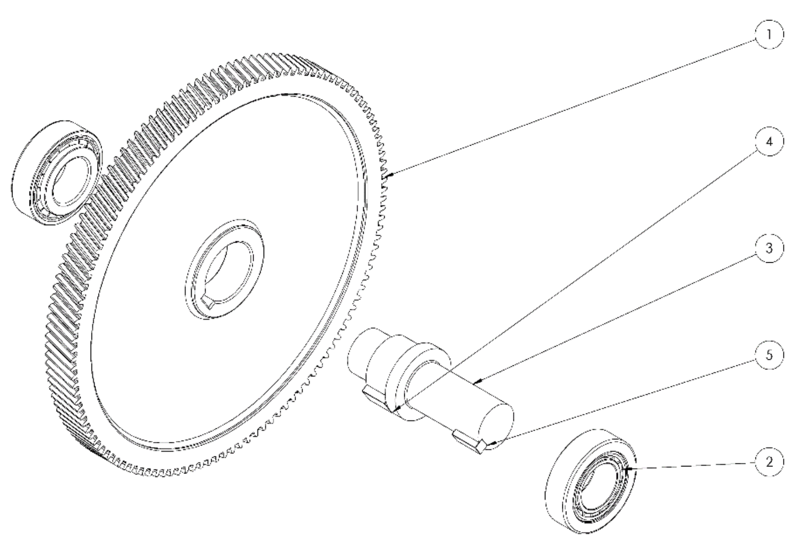
ITEM NO.	PART NUMBER	DESCRIPTION	QTY.
6	Right Intermediate endcap w fasteners		1
	Right Side Larg Cap		1
	Secondary cap		1
	B18.3.1M - 5 x 0.8 x 16 Hex SHCS -- 16NHX		4
7	B18.3.1M - 6 x 1.0 x 30 Hex SHCS -- 30NHX		4
8	B18.3.1M - 5 x 0.8 x 12 Hex SHCS -- 12NHX		4
9	Input Output Endcap Assembly		2
	1737N45		1
	Primary input cap		1
	B18.3.1M - 5 x 0.8 x 25 Hex SHCS -- 25NHX		4
	72mm Oring	72mm Oring	1
10	45975K32		1
11	Housing Top Draft		1
12	Top Plate gasket		1
13	B18.6.7M - M4 x 0.7 x 16 Type I Cross Recessed FHMS --16N		14
14	50785K231		1
15	72mm Oring	72mm Oring	2
16	90mm Oring	90mm Oring	1
17	Shim		3
18	Shim		1
19	Key 9x14mm		1

TITLE: EBOM ASSEMBLY

SIZE B DWG. NO. 2 REV

SCALE: 1:5 WEIGHT: SHEET 2 OF 2

Figure 33. Bill of materials



ITEM NO.	PART NUMBER	DESCRIPTION	QTY.
1	Large Gear DRAFT		1
2	32207		2
3	Input output Shaft		1
4	Key 14X9MM		1
5	Small key 10x8		1

TITLE: INPUT SHAFT EXPLODED VIEW

SIZE B DWG. NO. REV A

SCALE: 1:2 WEIGHT: SHEET 1 OF 1

Figure 34. Input shaft assembly drawing

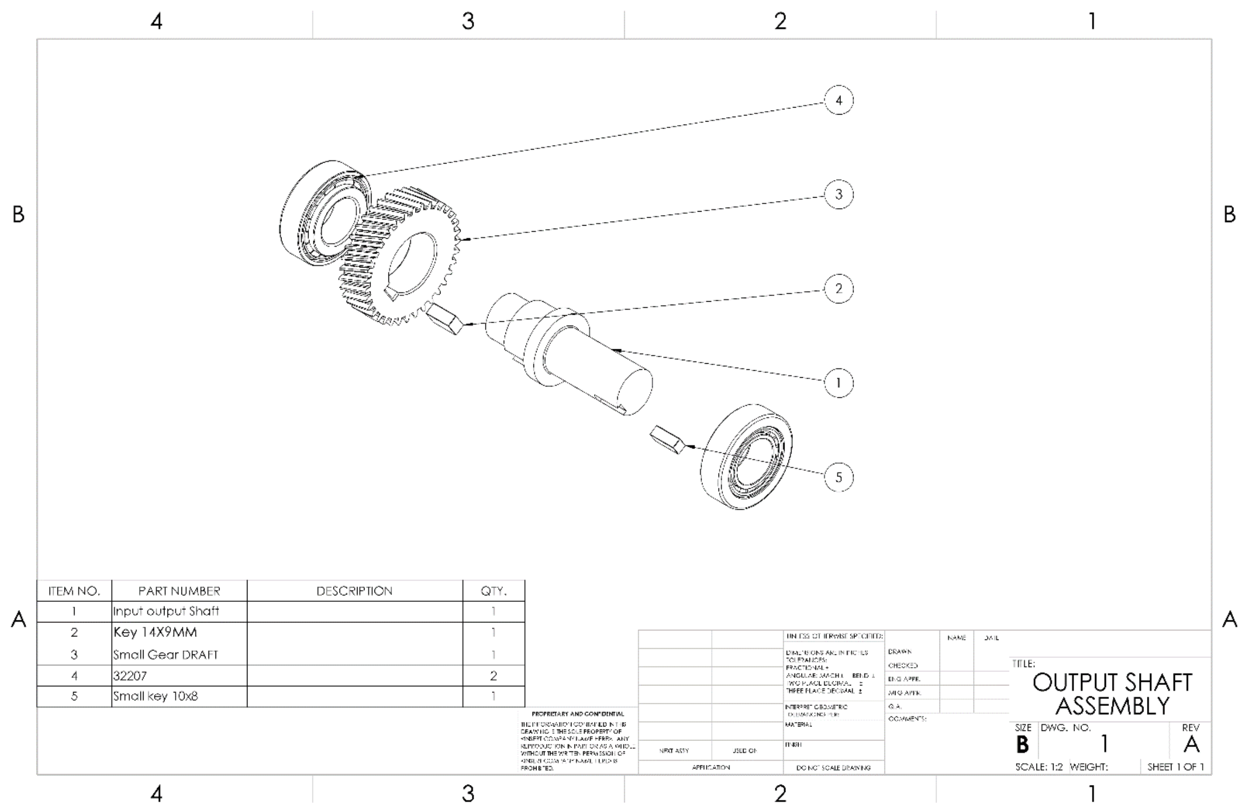
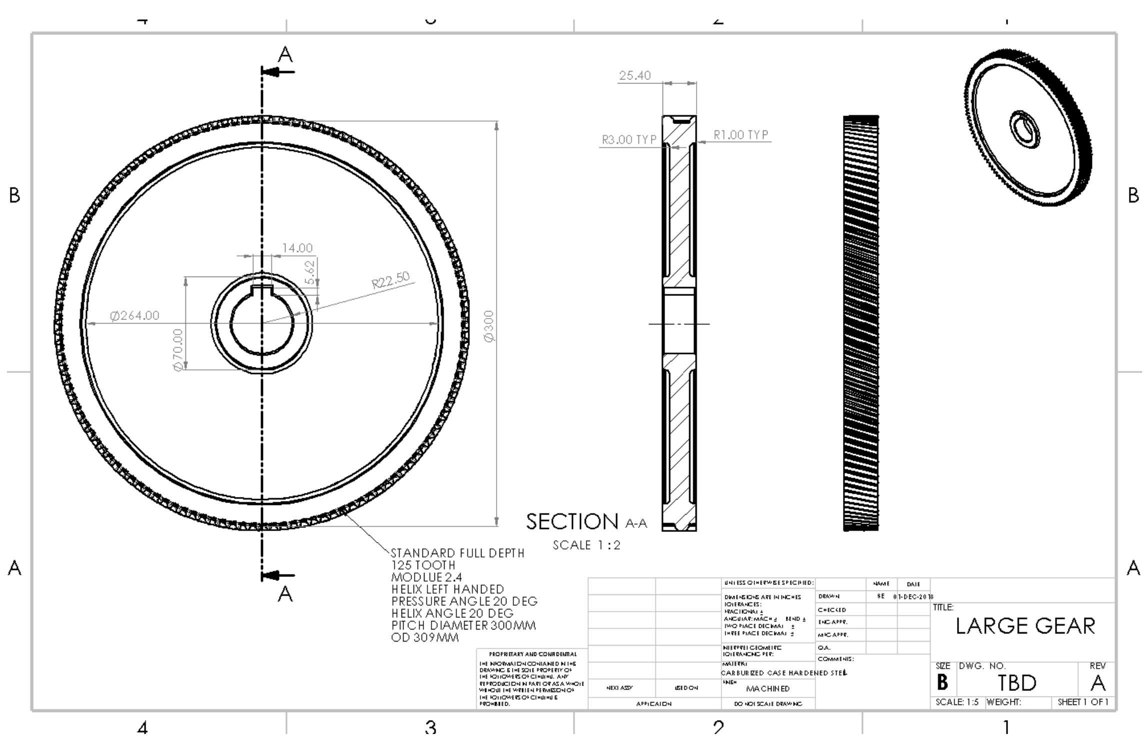


Figure 35. Output shaft assembly drawing



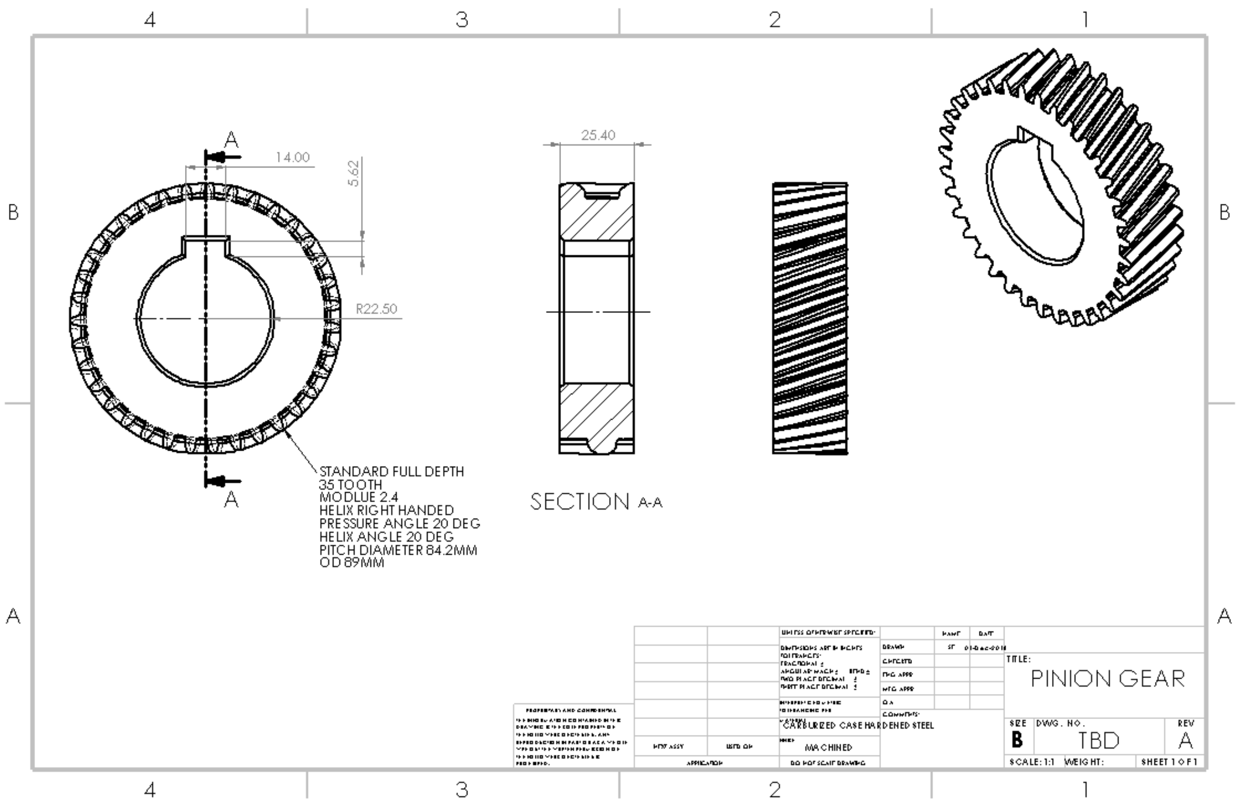


Figure 36. Intermediate shaft assembly drawing

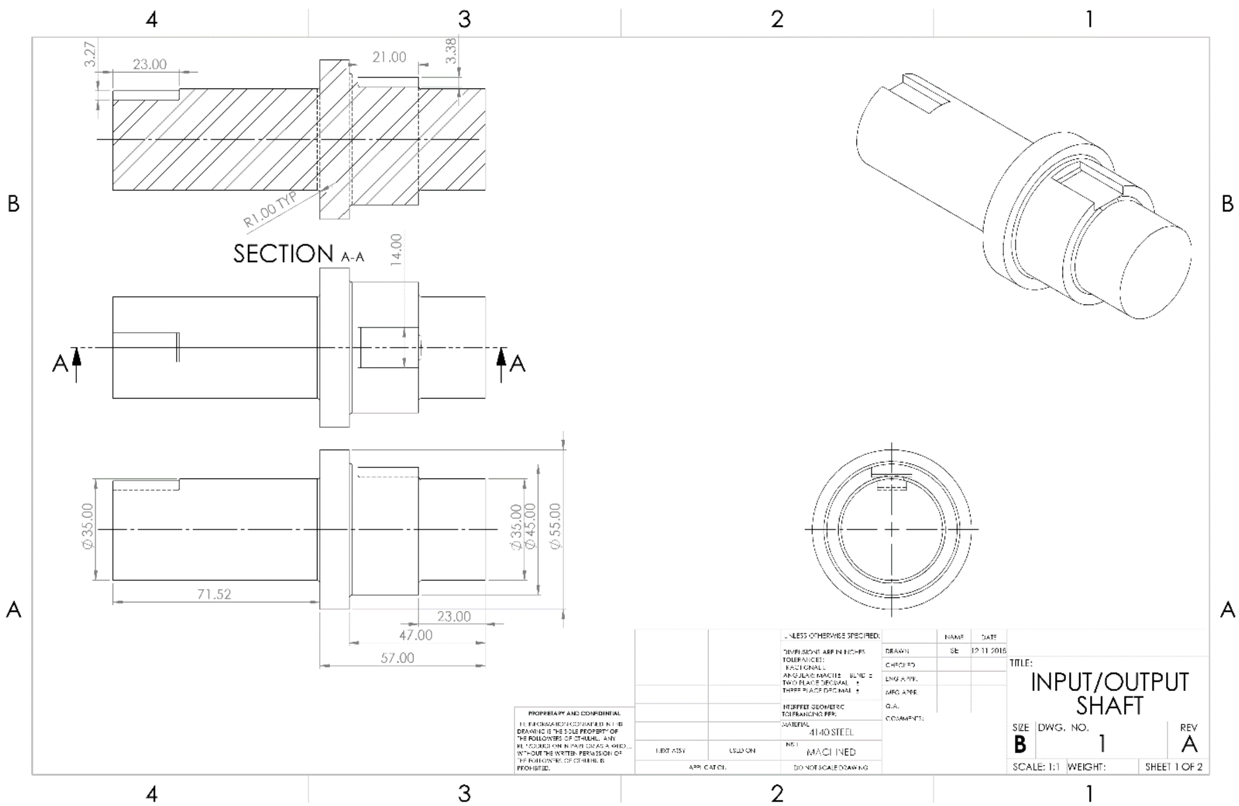


Figure 37. Input/output shaft drawing

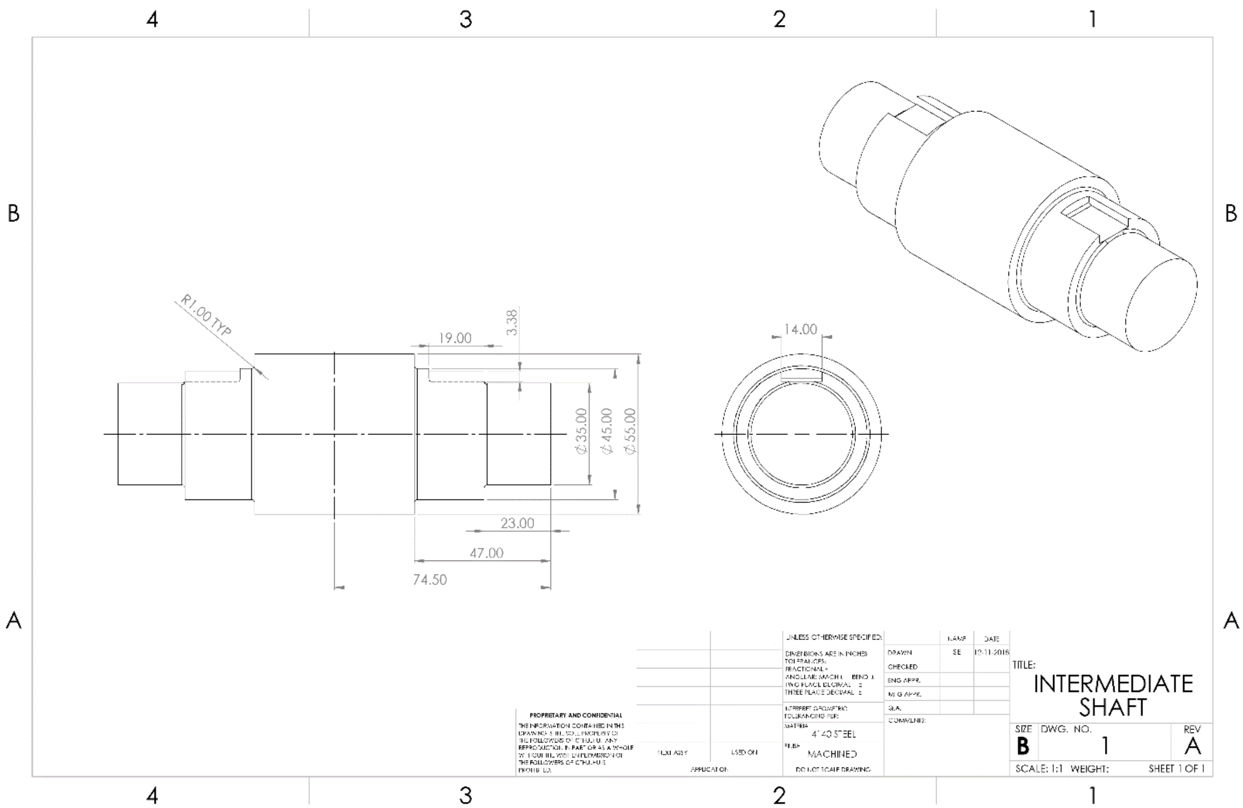


Figure 38. Intermediate shaft assembly drawing

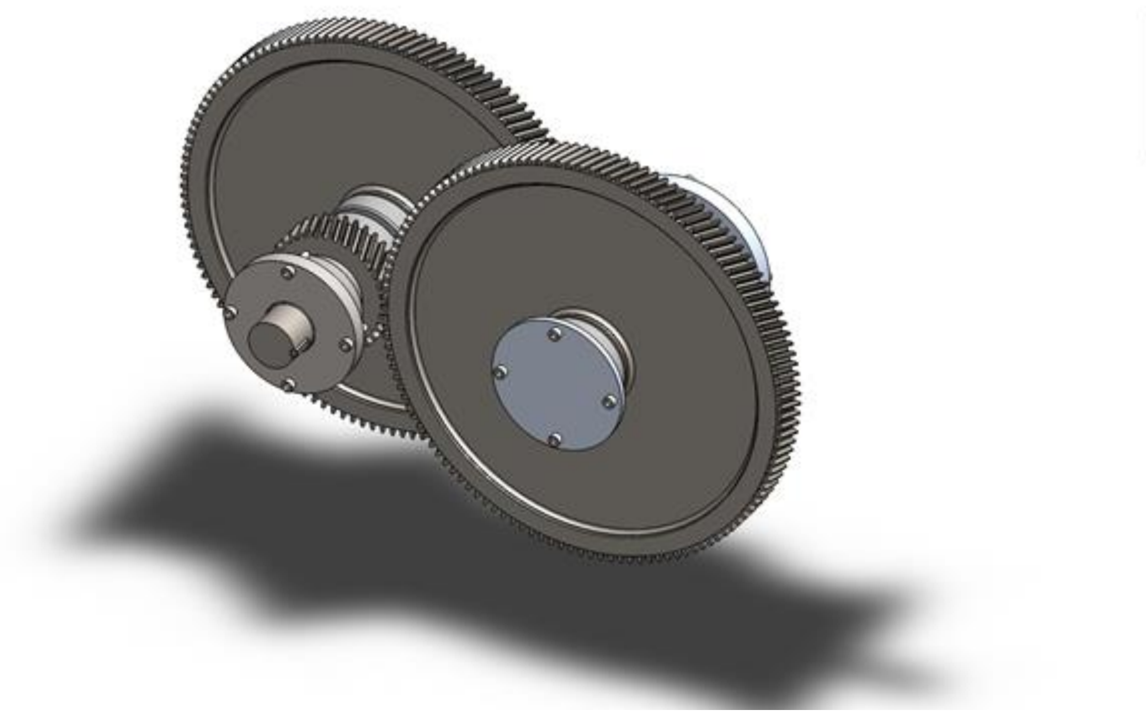


Figure 39. Isometric view of gear train assembly

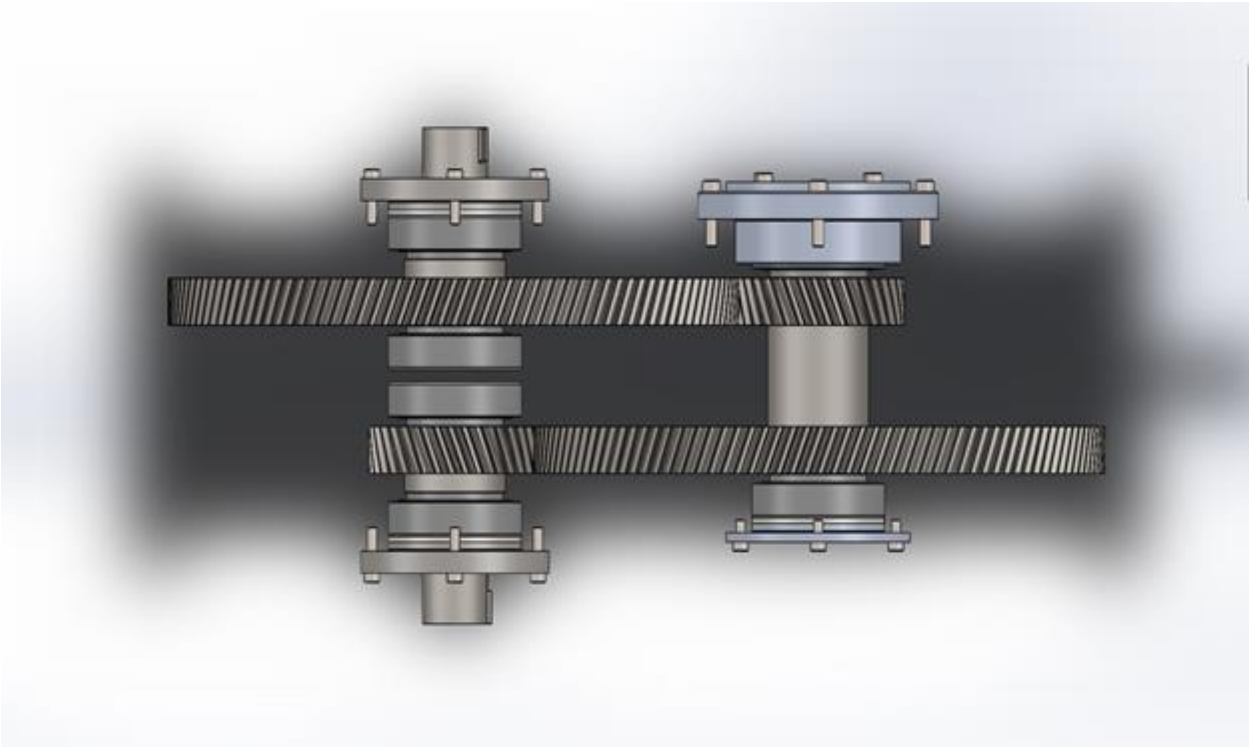


Figure 40. Top view of gear train assembly

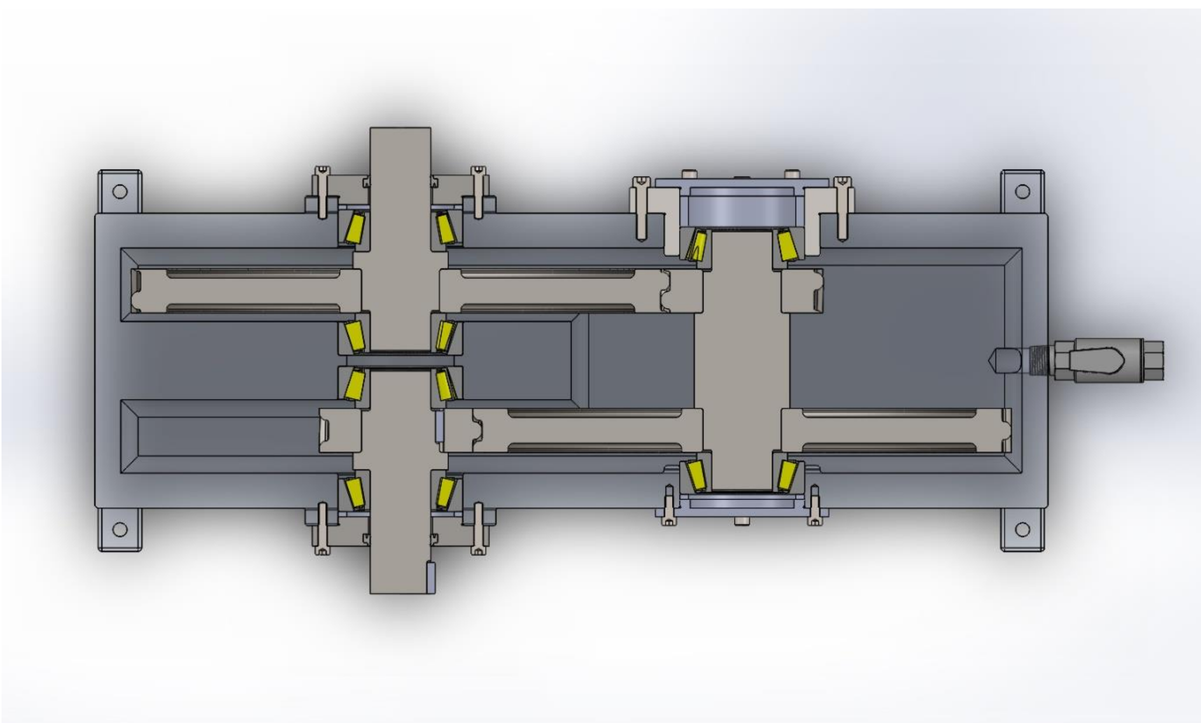


Figure 41. Top section view of gear train assembly

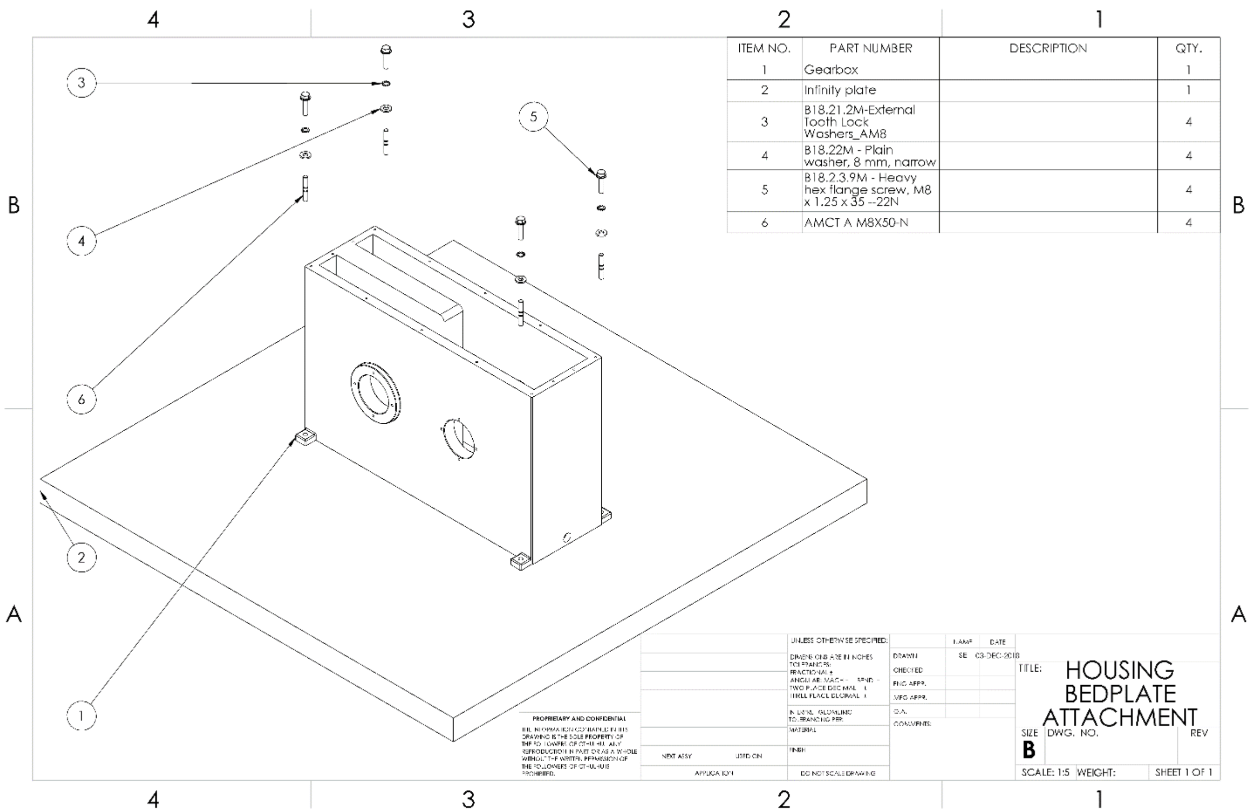


Figure 42. Housing mounting to infinitely strong bedplate

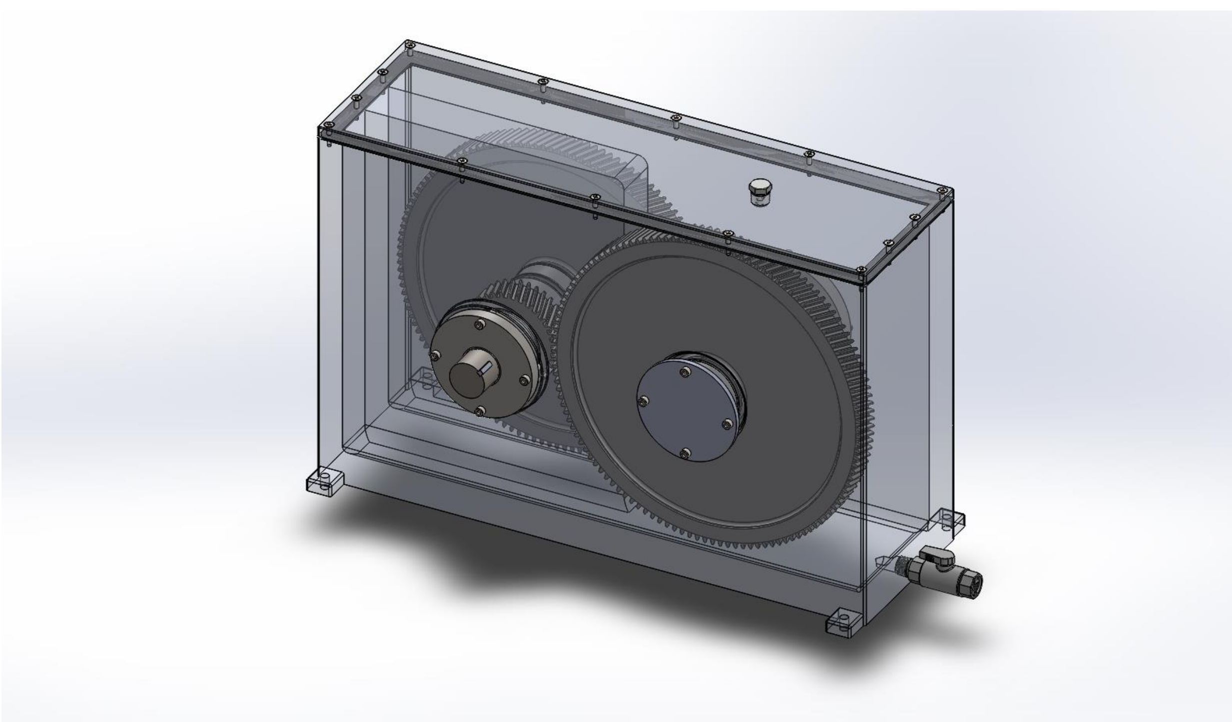


Figure 43. Isometric view of entire assembly

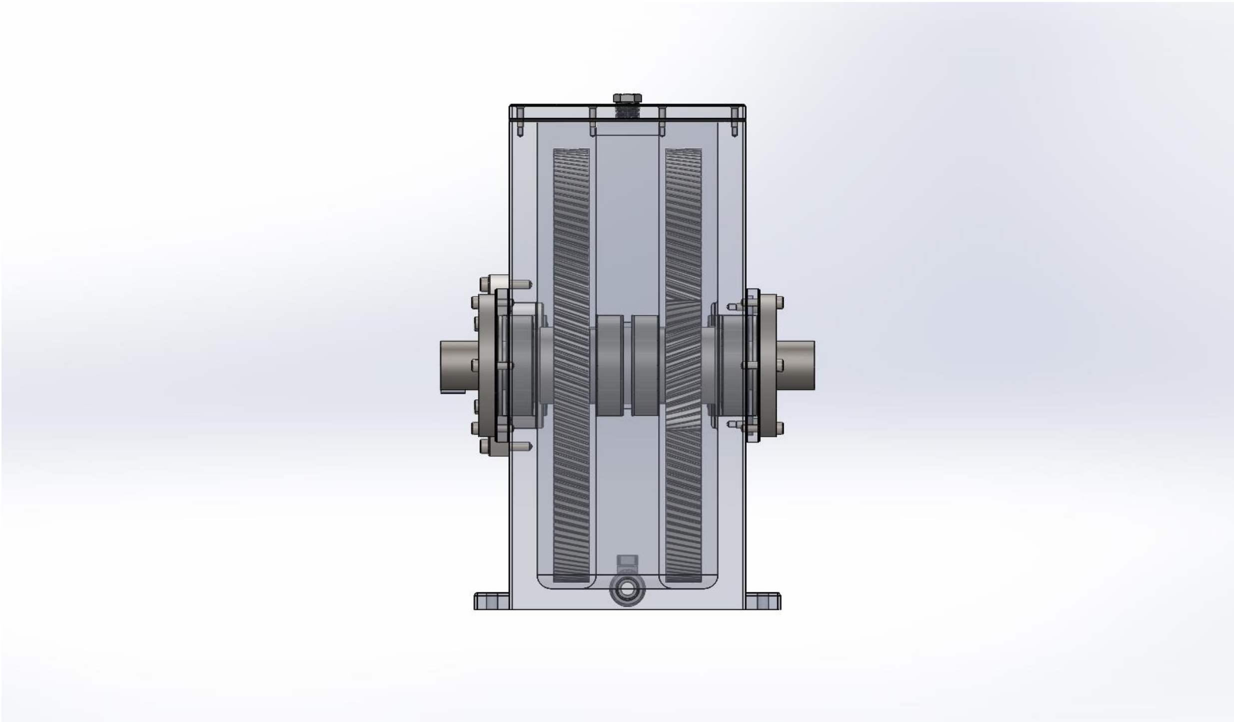


Figure 44 Right view of entire assembly

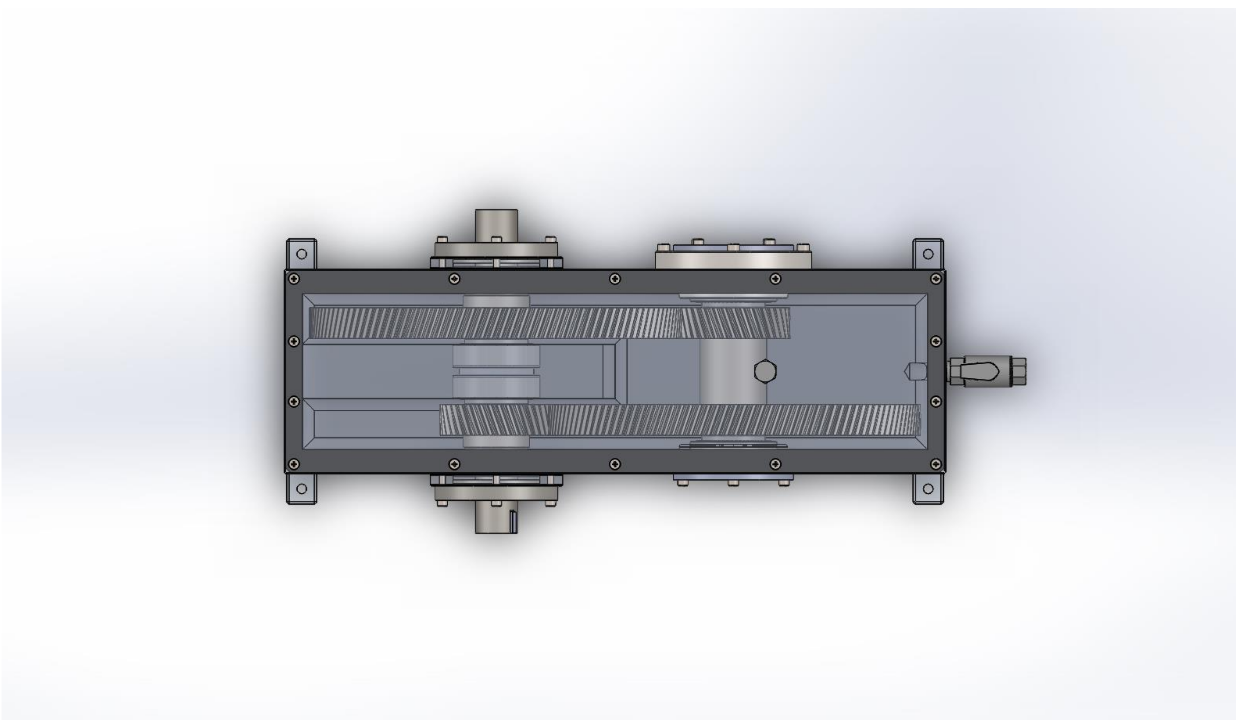


Figure 45. Top view of entire assembly

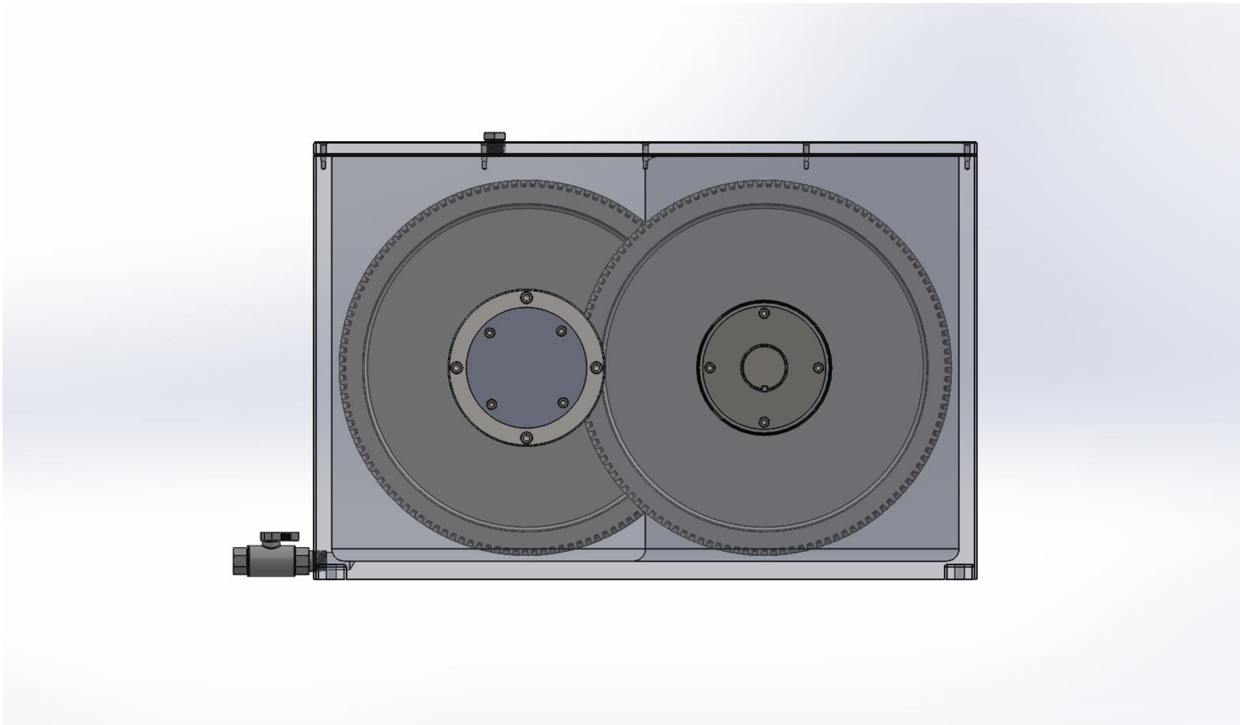


Figure 46. Back view of entire assembly

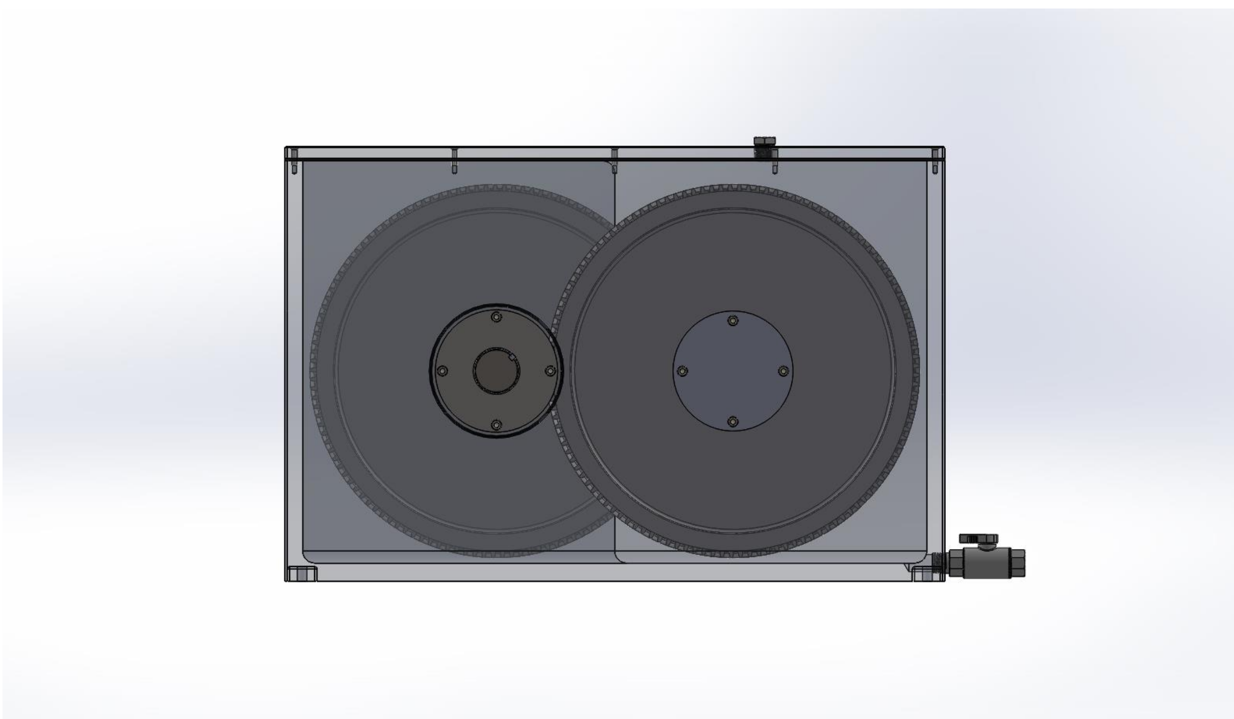


Figure 47. Front view of entire assembly

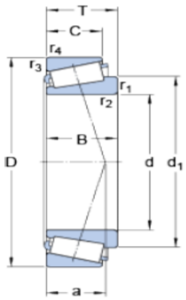
SKF

32207

Popular item
SKF Explorer

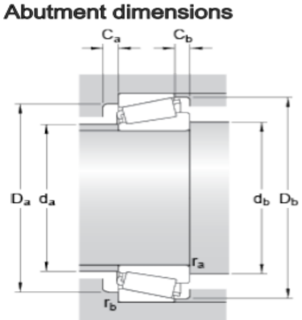
Dimension series	3DC

Dimensions



d		35	mm
D		72	mm
T		24.25	mm
d_1	≈	52.4	mm
B		23	mm
C		19	mm
$r_{1,2}$	min.	1.5	mm
$r_{3,4}$	min.	1.5	mm
a		17.442	mm

Abutment dimensions



d_a	max.	43	mm
d_b	min.	43.5	mm
D_a	min.	61	mm
D_a	max.	64.5	mm
D_b	min.	67	mm
C_a	min.	3	mm
C_b	min.	5	mm
r_a	max.	1.5	mm
r_b	max.	1.5	mm

Calculation data

Basic dynamic load rating	C	81.2	kN
Basic static load rating	C_0	78	kN
Fatigue load limit	P_u	8.5	kN
Reference speed		8000	r/min
Limiting speed		9500	r/min
Calculation factor	e	0.37	
Calculation factor	Y	1.6	
Calculation factor	Y_0	0.9	

Mass

Mass bearing	0.44	kg

Figure 48: Bearing datasheet

Appendix D: Free Body Diagrams and Loading Calculations

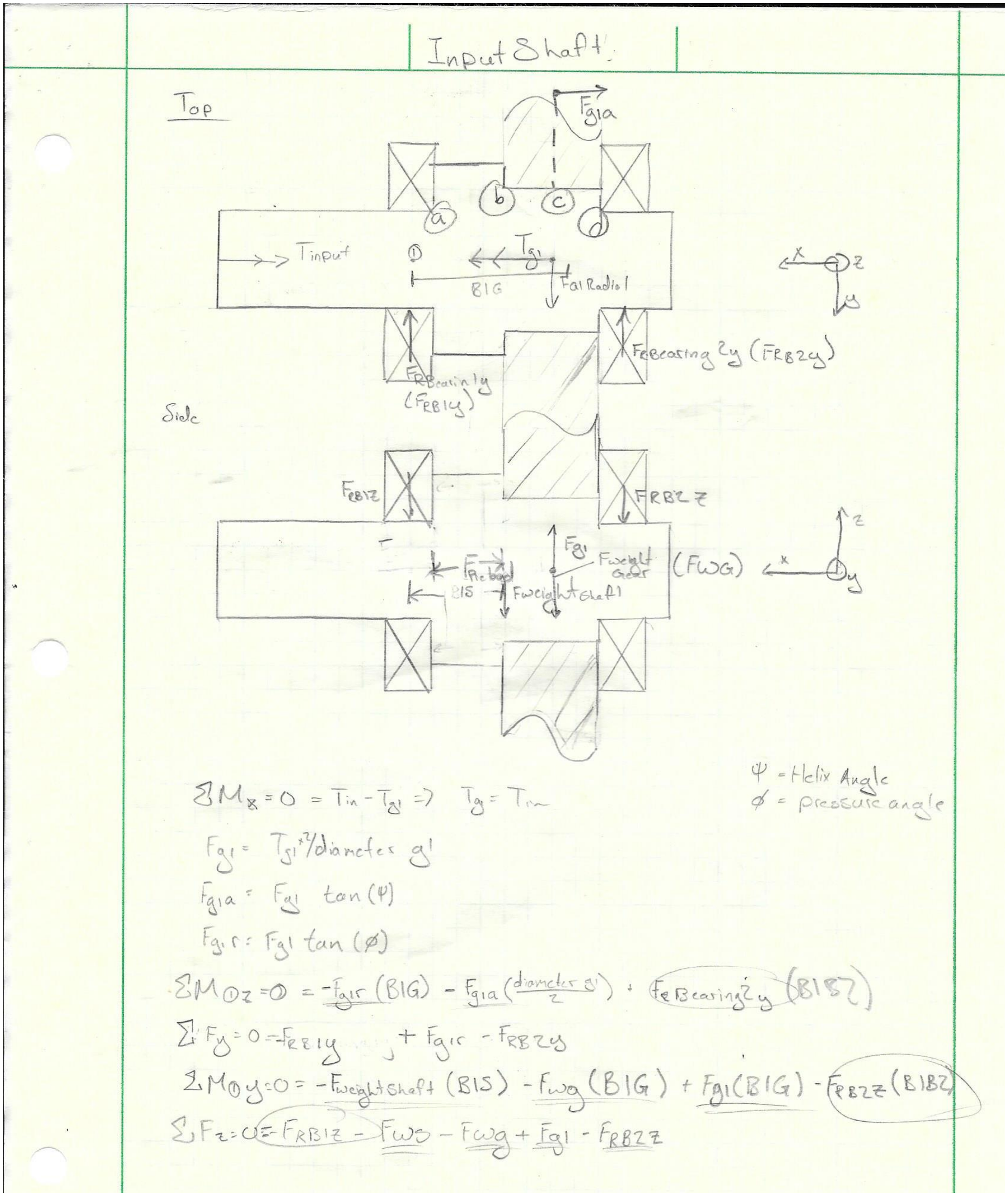
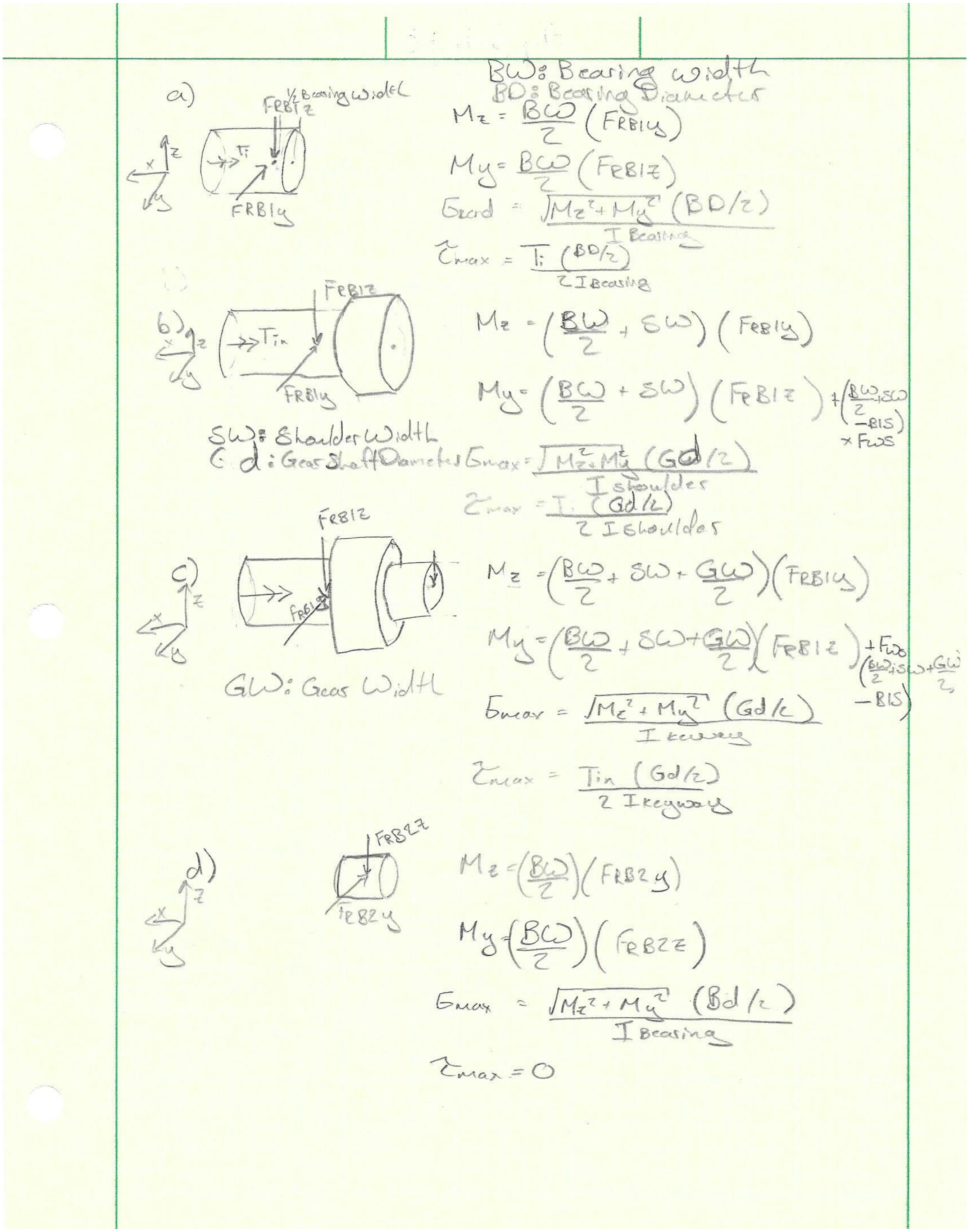


Figure 49. Input shaft free body diagram 1


BW: Bearing Width
BD: Bearing Diameter

a)

½ Bearing width

F_{RB1z}, T_i, F_{RB1y}

$$M_z = \frac{BW}{2}(F_{RB1y})$$

$$M_y = \frac{BW}{2}(F_{RB1z})$$

$$\sigma_{card} = \frac{\sqrt{M_z^2+M_y^2}\,(BD/2)}{I_{Bearing}}$$

$$\tau_{max} = \frac{T_i\,(BD/2)}{2 I_{Bearing}}$$

b)

F_{RB1z}, T_{in}, F_{RB1y}

$$M_z = \left(\frac{BW}{2} + SW\right)(F_{RB1y})$$

$$M_y = \left(\frac{BW}{2} + SW\right)(F_{RB1z}) + \left(\frac{BW}{2} + SW - B1S\right) \times F_{WS}$$

SW: Shoulder Width
Gd: Gear Shaft Diameter

$$\sigma_{max} = \frac{\sqrt{M_z^2+M_y^2}\,(Gd/2)}{I_{shoulder}}$$

$$\tau_{max} = \frac{T_i\,(Gd/2)}{2 I_{shoulder}}$$

c)

F_{RB1z}, F_{RB1y}

$$M_z = \left(\frac{BW}{2} + SW + \frac{GW}{2}\right)(F_{RB1y})$$

$$M_y = \left(\frac{BW}{2} + SW + \frac{GW}{2}\right)(F_{RB1z}) + F_{WS}\left(\frac{BW}{2} + SW + \frac{GW}{2} - B1S\right)$$

GW: Gear Width

$$\sigma_{max} = \frac{\sqrt{M_z^2+M_y^2}\,(Gd/2)}{I_{keyway}}$$

$$\tau_{max} = \frac{T_{in}\,(Gd/2)}{2 I_{keyway}}$$

d)

F_{RB2z}, F_{RB2y}

$$M_z = \left(\frac{BW}{2}\right)(F_{RB2y})$$

$$M_y = \left(\frac{BW}{2}\right)(F_{RB2z})$$

$$\sigma_{max} = \frac{\sqrt{M_z^2+M_y^2}\,(Bd/2)}{I_{Bearing}}$$

$$\tau_{max} = 0$$

Figure 50. Input shaft free body diagram 2

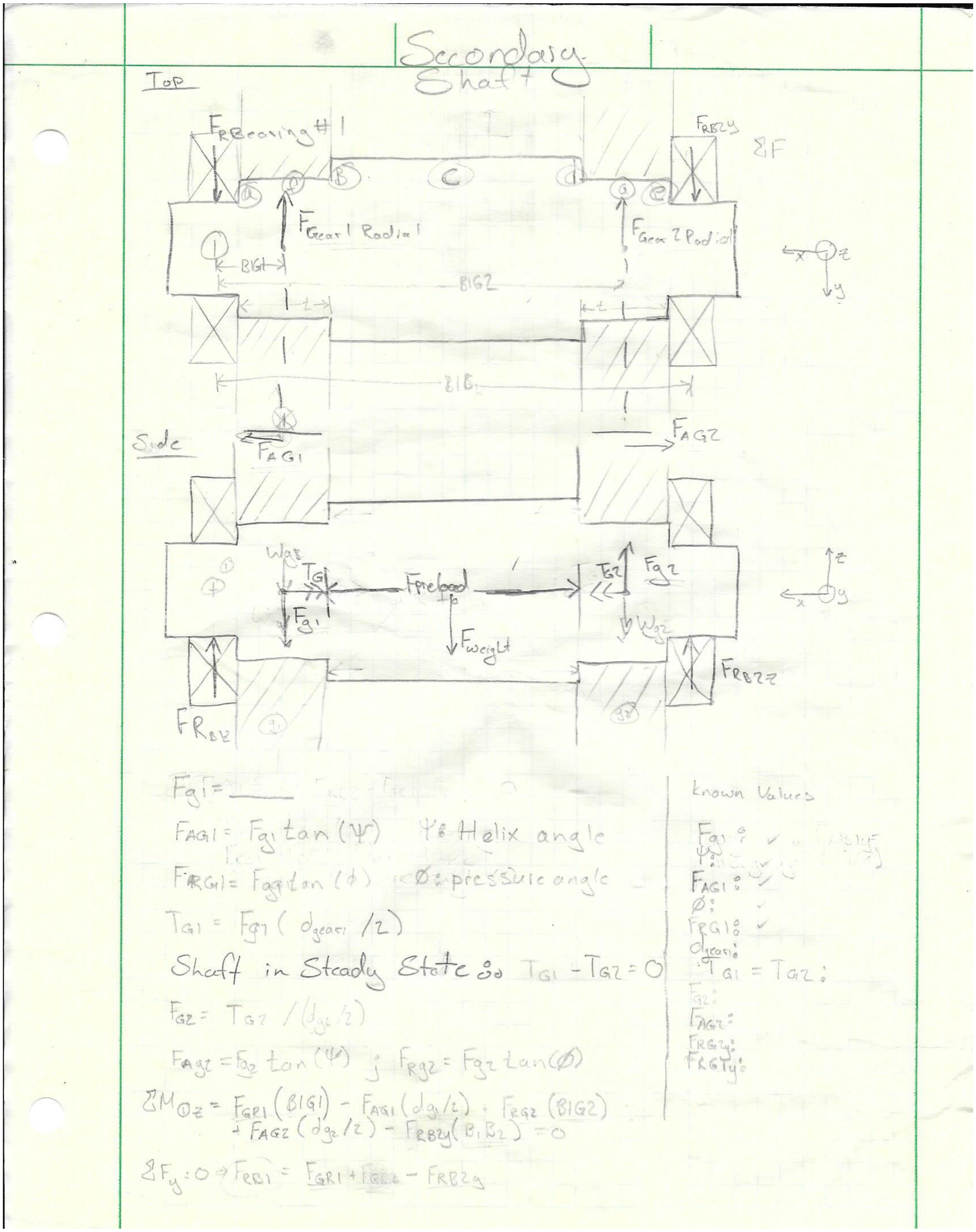


Figure 51. Intermediate shaft free body diagram 1

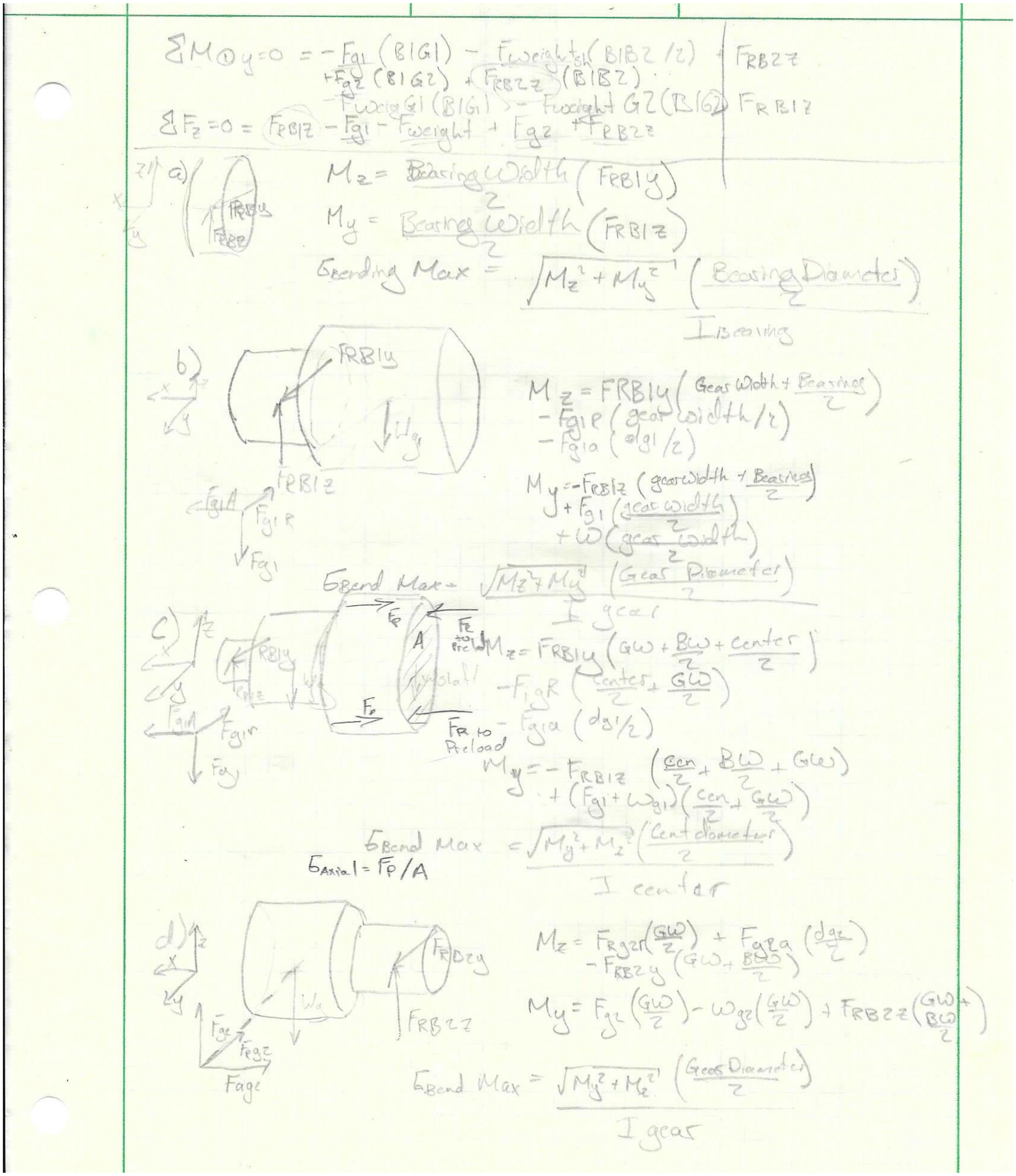


Figure 52. Intermediate shaft free body diagram 2

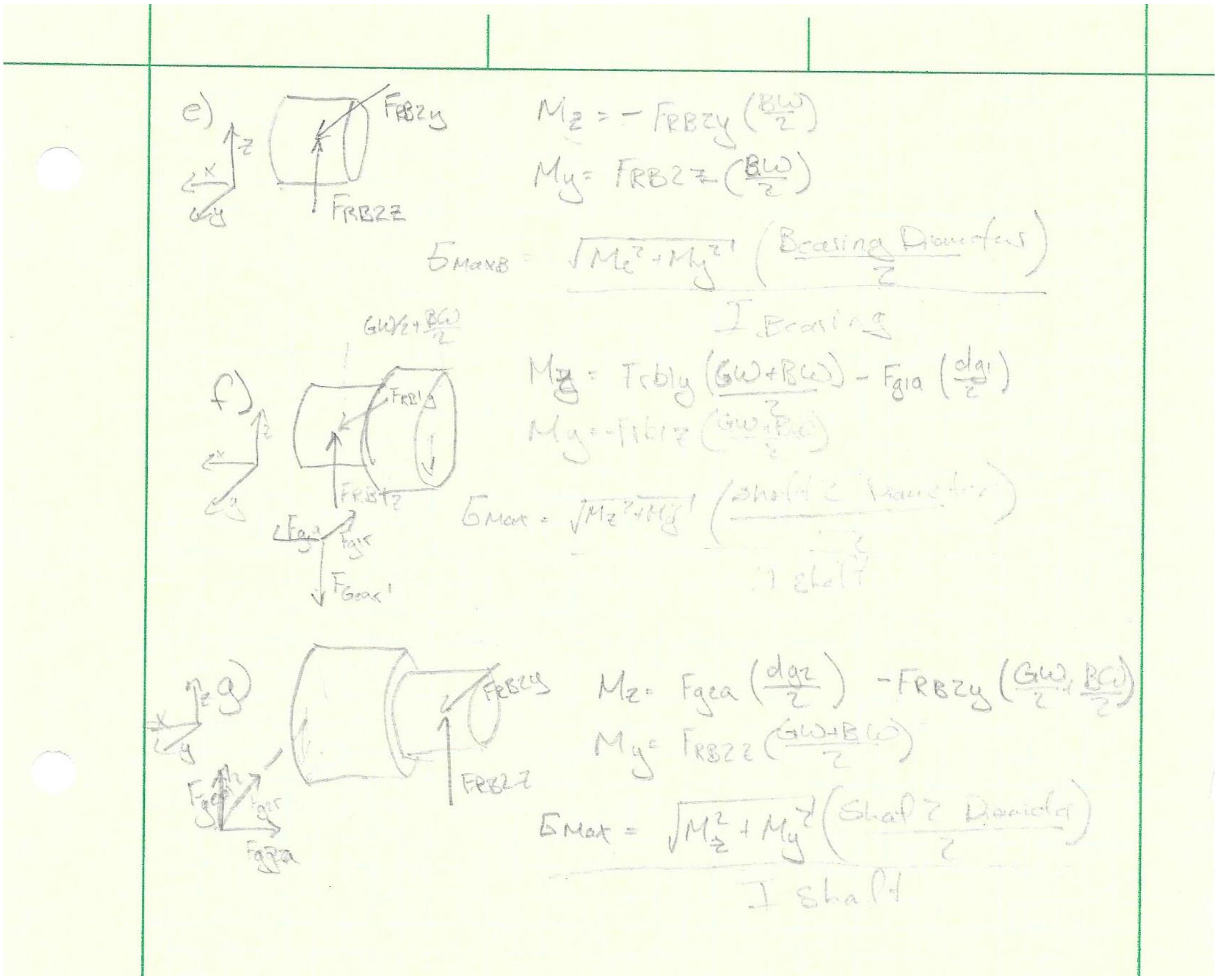
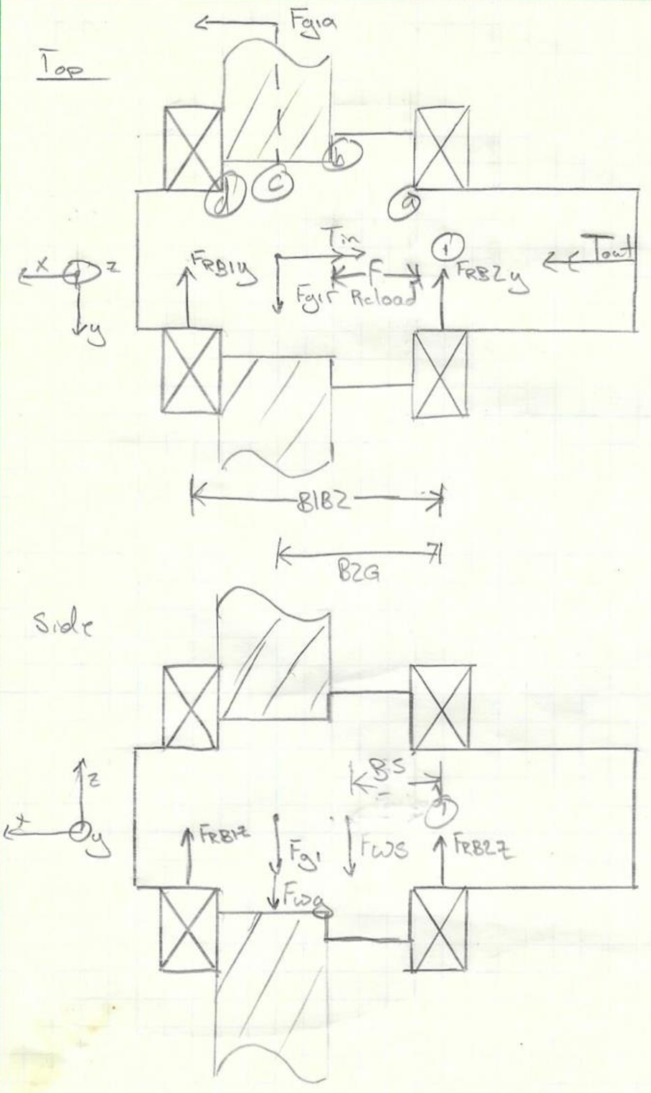


Figure 53. Intermediate shaft free body diagram 3

Output Shaft



$T_i = T_o$

$F_{g1} = T_i / (d_{gear}/2)$

$F_{g1a} = F_{g1} \tan(\psi)$

$F_{g1r} = F_{g1} \tan(\phi)$

ψ Helix angle
ϕ Pressure angle

$\sum M_{①z} = -F_{RB1y}(B1B2) + F_{g1r}(B2G) + F_{g1a}\left(\frac{GD}{2}\right)$

$\sum F_y = 0 = F_{RB1y} + F_{RB2y} - F_{g1R} = 0$

$\sum M_{①y} = 0 = -F_{RB1z}(B1B2) + (F_{g1} + F_{wg}) B2G + F_{ws}(BS)$

$\sum F_z = F_{RB1z} + F_{RB2z} - F_{wg} - F_{ws} - F_{g1} = 0$

Figure 54. Output shaft free body diagram 1

a) $M_z = \frac{BW}{2}(F_{RB2y})$ $\quad \sigma_{max} = \frac{\sqrt{M_z^2 + M_y^2} * (Bd/2)}{I_{Bearing}}$

$M_y = \frac{BW}{2}(F_{RB2z})$ $\quad \tau_{max} = \frac{T_{out}(Bd/2)}{2 I_{Bearing}}$

b) $M_z = \left(\frac{BW}{2} + SW\right)(F_{RB2y})$ $\quad \sigma_{max} = \frac{\sqrt{M_z^2 + M_y^2}(GW/2)}{I_{shoulder}}$

$M_y = \left(\frac{BW}{2} + SW\right)(F_{RB2z})$ $\quad \tau_{max} = \frac{T_{out}(GW/2)}{2 I_{shoulder}}$

c) $M_z = \left(\frac{BW}{2} + SW + \frac{GW}{2}\right)(F_{RB2y}) + F_{g1a}\left(Gd/2\right)$

$M_y = \left(\frac{BW}{2} + SW + \frac{GW}{2}\right) F_{RB2z} - \left(\frac{BW}{2} + SW + \frac{GW}{2} - BS\right) F_{WS}$

$$\sigma_{max} = \frac{\sqrt{M_z^2 + M_y^2}(Bd/2)}{I_{keyway}}$$

$$\tau_{max} = \frac{T_{in}(Gd/2)}{2 I_{keyway}}$$

d) $M_z = \left(\frac{BW}{2}\right) F_{RB1y}$ $\quad \sigma_{max} = \frac{\sqrt{M_x^2 + M_y^2}\left(\frac{Bd}{2}\right)}{I_{Bearing}}$

$M_y = \frac{BW}{2}(F_{RB1z})$ $\quad \tau_{max} = 0$

Figure 55. Output shaft free body diagram 2